Άσκηση 3.54

1) $a = \frac{0,6-0,1}{6} = 0,0833 \Rightarrow a = 0,08314\,rad$

$P(x) = P(0)\,[1-(\mu\Sigma a + Kx)]$

$\frac{P(x)}{P(0)} = 1-(\mu\Sigma a + Kx)$

$\mu = 0,25 \quad , K = 0,8\cdot10^{-3}$

B_L

$\Sigma a_1 = 0 \quad , x = 6m$

$P_{B,L} = P_{0,0}\,(1-0,8\cdot10^{-3}\cdot 6)$

$P_{B,L} = P_{0,0}\,(1-4,8\cdot10^{-3})$

$\frac{P_{B,L}}{P_{0,0}} = 0,9952$

B_R

$\tan a_2 = \frac{1}{\frac{9}{2}} = 0,22222 \Rightarrow a_2 = 0,21867\,rad, \; x_2 = 9m$

$\Sigma a = a_1 = a + \frac{a_2}{2} = 0,08314 + 0,21867 = 0,30181$

$\Sigma x = x_1 + 0 = 6m$

$\mu\Sigma a + Kx = 0,25\cdot 0,30181 + 0,8\cdot10^{-3}\cdot 6 = 0,08025$

$\frac{P_{B,R}}{P_{0,0}} = 1-0,08025 = 0,9197$

Γ.

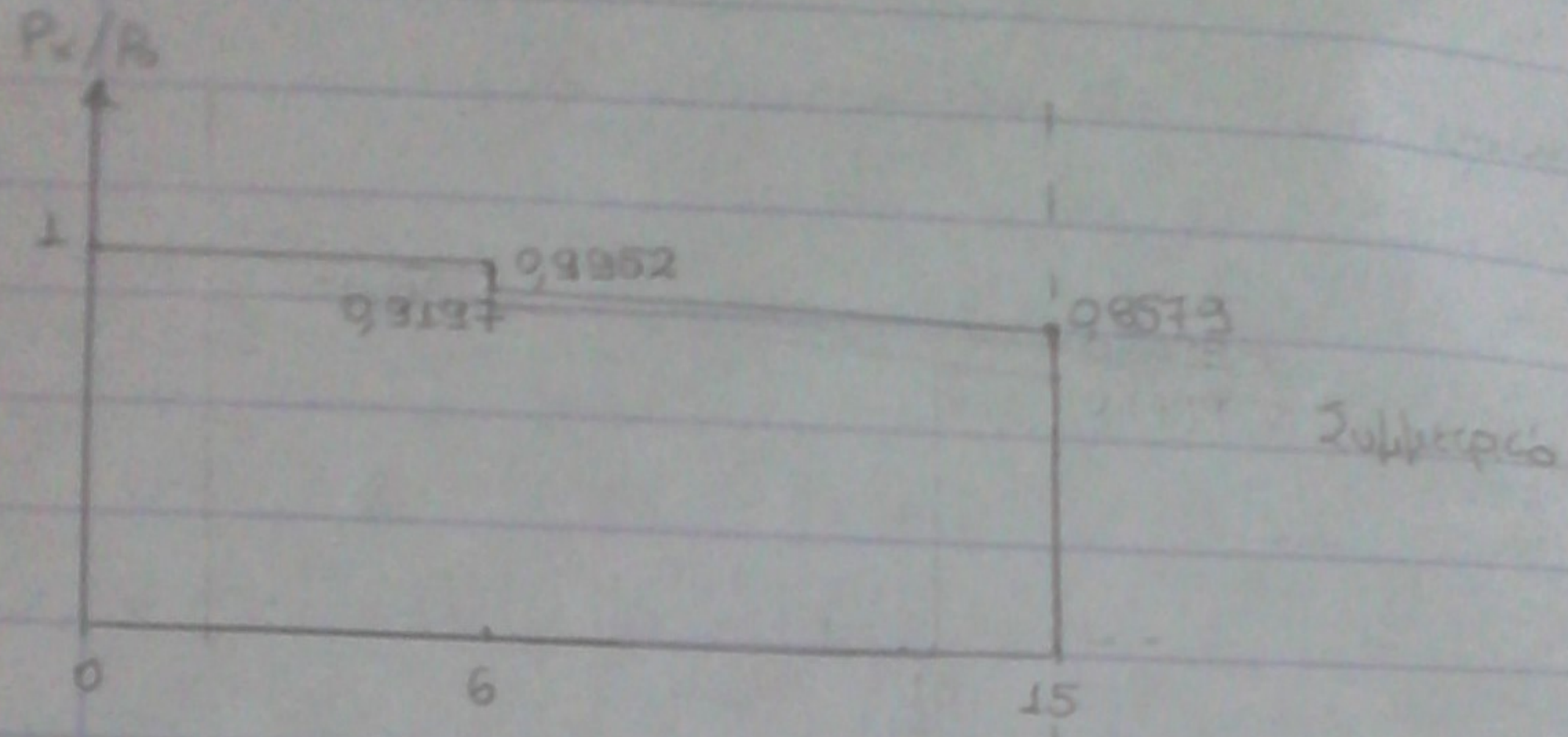
$a_2 = 0,21867$

$\Sigma a = a_1 + a_2 = 0,30181 + 0,21867 = 0,52048$

$x_3 = 9m$

$\mu\Sigma a + Kx = 0,25\cdot 0,52048 + 0,8\cdot10^{-3}(6+9) = 0,1421$

$$\frac{P_0}{P_{\infty}} = 1 - 91421 = 0,8579$$



2) $\sigma_{p0} = 1150 MPa = 115 kN/cm^2$

$A_p = 21 cm^2$

$E_p = 195 Gpa = 195\,10^3 MPa = 19500 kN/cm^2$

$$\varepsilon_{p0} = \frac{\sigma_{p0}}{E_p} = \frac{115}{19500} = 5,897 \cdot 10^{-3}$$

$$\Delta l_A = \Delta l_E = \int_0^x \varepsilon_p(x)\,dx = \int_0^x \frac{\sigma_p(x)}{E_p}\,dx = \int_0^x \frac{P_0(x)}{A_p E_p}\,dx$$

$$= \frac{P_0(0)}{A_p E_p} \int_0^x \frac{P_0(x)}{P_0(0)}\,dx$$

$$= \varepsilon_{p0} \int_0^x \frac{P_0(x)}{P_0(0)}\,dx$$

$$\int_0^x \frac{P_0(x)}{P_0(0)}\,dx = \frac{1+0,9952}{2}\,6 + \frac{0,9198+0,8579}{2}\,9 = 6,955$$

$\Delta l_1 = \Delta l_6 = 5{,}897 \cdot 10^{-3} \cdot 13{,}333 = 0{,}0925 m = 92{,}5 mm$

3. Μονοπλευρη επιβολη προεντασεως

$\Delta_{,2}$

$a_4 = a_3 = 0{,}21867 \, rad$

$x_4 = 9m$

$\Sigma a = 0{,}08314 + 3 \cdot 0{,}21867 = 0{,}73915 \, rad$

$\Sigma x = 15m + 9m = 24m$

$\mu \Sigma a + k \Sigma x = 0{,}25 \cdot 0{,}73915 + 0{,}8 \cdot 10^{-3} \cdot 24 = 0{,}2040$

$\frac{P_{0\Delta 2}}{P_{0,0}} = 1 - 0{,}2040 = 0{,}7960$

$\Delta_{,2}$

$a_5 = a_4 = 0{,}21867 \, rad$

$x_5 = 0m$

$\Sigma a = 0{,}08314 + 4 \cdot 0{,}21867 = 0{,}95782 \, rad$

$\Sigma x = 24m$

$\mu \Sigma a + k \Sigma x = 0{,}25 \cdot 0{,}95782 + 0{,}8 \cdot 10^{-3} \cdot 24 = 0{,}2587$

$\frac{P_{0\Delta 2}}{P_{0,0}} = 1 - 0{,}2587 = 0{,}74135$

E

$a_6 = a_1 = 0{,}08314$

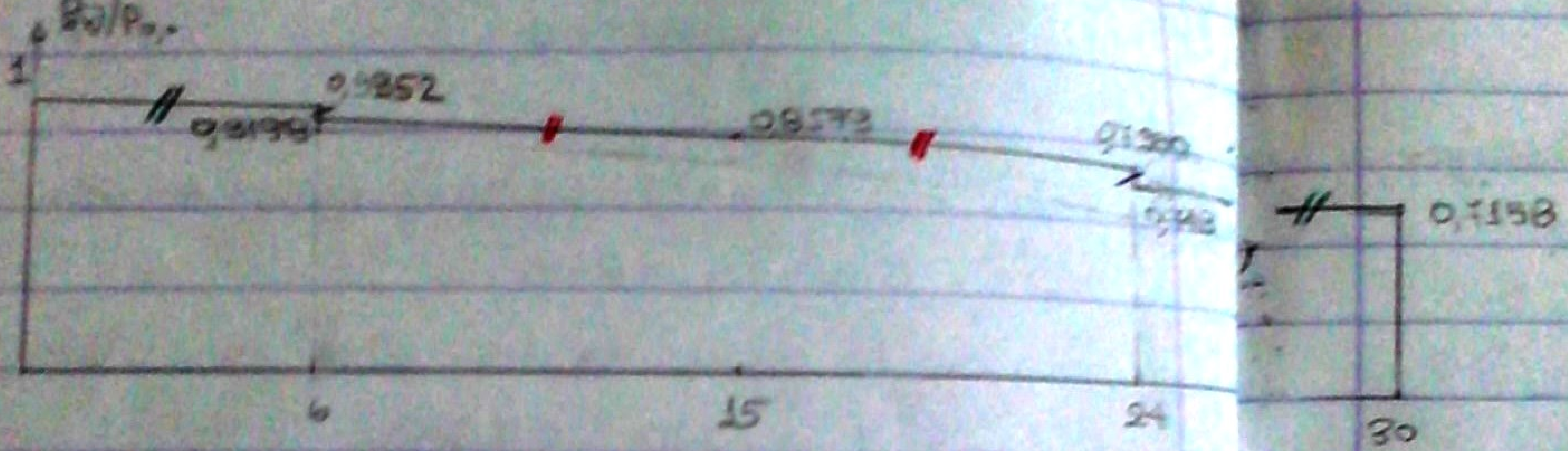
$x = 30m$

$\Sigma\alpha = 2\cdot 0,0834 + 2\cdot 0,2967 = 1,04096 \text{ rad}$

$\Sigma x = 30m$

$\mu\Sigma\alpha + k\Sigma x = 0,25\cdot 1,04096 + 9,8\cdot 10^{-3}\cdot 30 = 0,28424$

$$\frac{P_{x,e}}{P_{x,p}} = 1 - 0,28424 = 0,7158$$



Διάγραμμα κινήσεων για

κατανομή τάσεων

στο αριστερό άκρο

$$\Delta l_A = \int_0^x \varepsilon_p(x)dx = \int_0^x \frac{\sigma_p(x)}{E_p}dx = \int_0^x \frac{P_o(x)}{E_p A_p}dx = \int_0^x \frac{P_o(x)}{P_o(0)}\frac{P_o(0)}{E_p A_p}dx$$

$$= \frac{P_o(0)}{E_p A_p}\int_0^x \frac{P_o(x)}{P_o(0)}dx = \varepsilon_{p,0}\int_0^x \frac{P_o(x)}{P_o(0)}dx$$

$$\int_0^x \frac{P_o(x)}{P_o(0)}dx = 0,5\big[(1+0,9852)\cdot 6 + (0,9198+0,8579)\cdot 9 + (0,8579+0,7360)\cdot 9 + (0,7418+0,7158)\cdot 6\big]$$

$$= 25,799$$

$$\Delta l_A = 5,897\cdot 10^{-3}\cdot 25,799 = 0,152\,m = 152\,mm$$