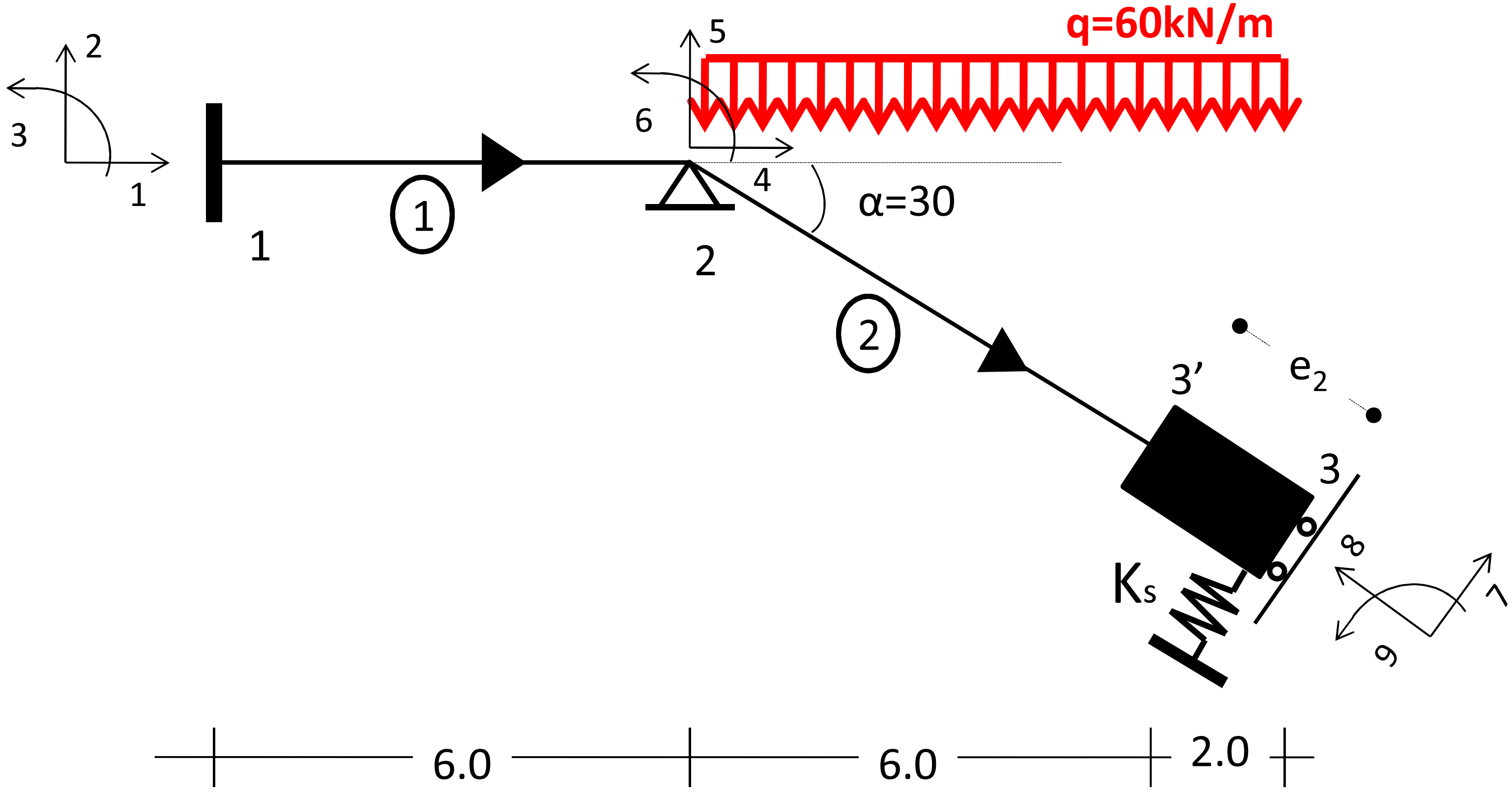
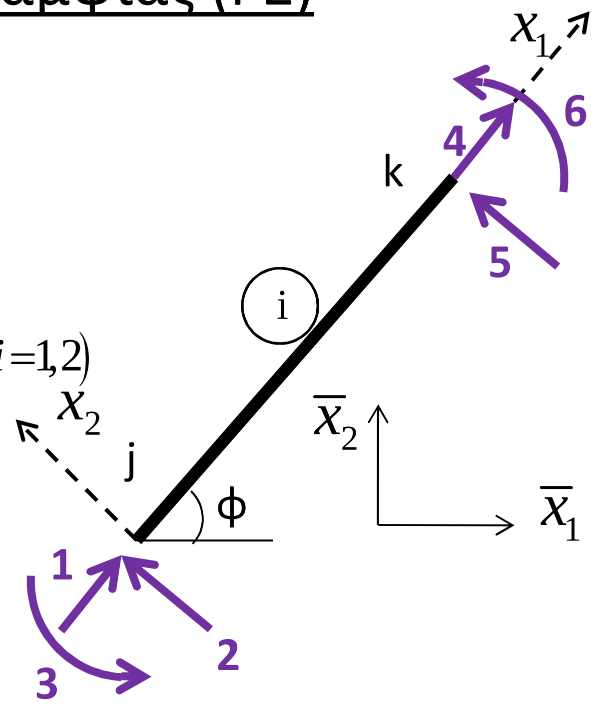
Άσκηση Πλαισίου II



Μόρφωση τοπικών μητρώων δυσκαμψίας (P2)

$$
\left[k^i\right]=\begin{bmatrix}
A_iE_i/L_i & 0 & 0 & -A_iE_i/L_i & 0 & 0 \\
0 & 12E_iI^i/L_i^3 & 6E_iI^i/L_i^2 & 0 & -12E_iI^i/L_i^3 & 6E_iI^i/L_i^2 \\
0 & 6E_iI^i/L_i^2 & 4E_iI^i/L_i & 0 & -6E_iI^i/L_i^2 & 2E_iI^i/L_i \\
-A_iE_i/L_i & 0 & 0 & A_iE_i/L_i & 0 & 0 \\
0 & -12E_iI^i/L_i^3 & -6E_iI^i/L_i^2 & 0 & 12E_iI^i/L_i^3 & -6E_iI^i/L_i^2 \\
0 & 6E_iI^i/L_i^2 & 2E_iI^i/L_i & 0 & -6E_iI^i/L_i^2 & 4E_iI^i/L_i
\end{bmatrix}
\quad (i=1,2)
$$

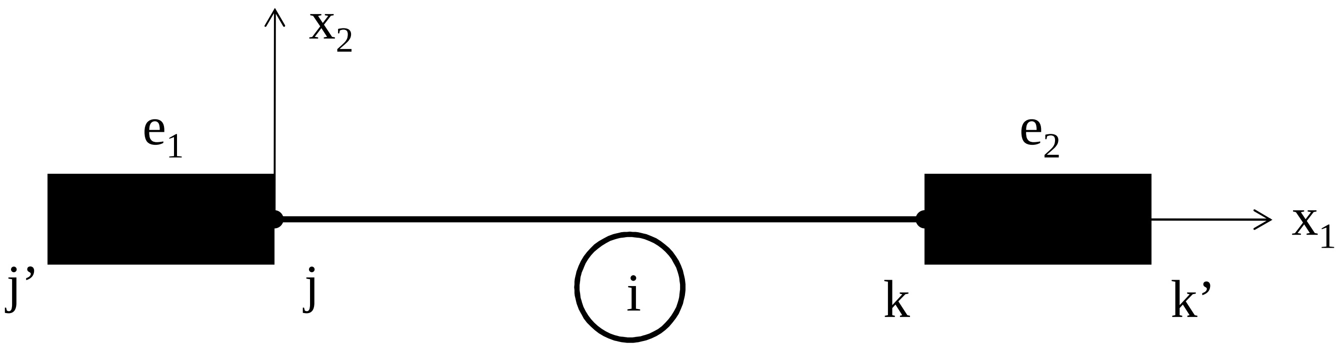


$$
\left[k^1\right]=EI\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix}
3.125 & 0 & 0 & -3.125 & 0 & 0 \\
0 & 0.055 & 0.166 & 0 & -0.055 & 0.166 \\
0 & 0.166 & 0.666 & 0 & -0.166 & 0.333 \\
-3.125 & 0 & 0 & 3.125 & 0 & 0 \\
0 & -0.055 & -0.166 & 0 & 0.055 & -0.166 \\
0 & 0.166 & 0.333 & 0 & -0.166 & 0.666
\end{bmatrix}
\end{array}
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}
$$

$$
\left[k^2\right]=EI\begin{array}{c}
\begin{array}{cccccc} 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\
\begin{bmatrix}
2.706 & 0 & 0 & -2.706 & 0 & 0 \\
0 & 0.036 & 0.125 & 0 & -0.036 & 0.125 \\
0 & 0.125 & 0.577 & 0 & -0.125 & 0.288 \\
-2.706 & 0 & 0 & 2.706 & 0 & 0 \\
0 & -0.036 & -0.125 & 0 & 0.036 & -0.125 \\
0 & 0.125 & 0.288 & 0 & -0.125 & 0.577
\end{bmatrix}
\end{array}
\begin{array}{c} \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array}
$$

Μόρφωση τοπικών μητρώων δυσκαμψίας (P2) λόγω εκκεντρότητας

$$\left[\bar{k}^i \right] = \left[e^i \right]^T \left[k^i \right] \left[e^i \right]$$

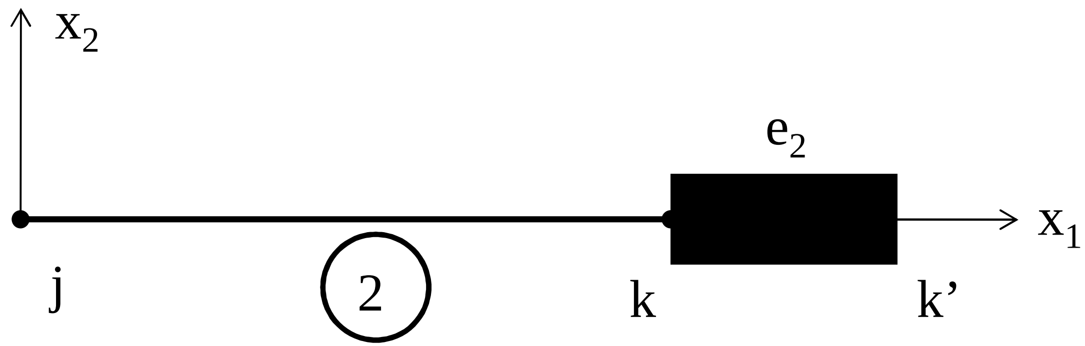


$$[e_1] = [I]$$

$$[e^i] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & & & \\ 0 & 1 & e_1 & & [0] & \\ 0 & 0 & 1 & & & \\ \hline & & & 1 & 0 & 0 \\ & [0] & & 0 & 1 & -e_2 \\ & & & 0 & 0 & 1 \end{array}\right] \quad \xRightarrow{e_1 = 0,\ e_2 = \frac{2}{\cos 30}} \quad [e^2] = \begin{bmatrix} 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & [0] & \\ 0 & 0 & 1 & & & \\ & & & 1 & 0 & 0 \\ & [0] & & 0 & 1 & -2.309 \\ & & & 0 & 0 & 1 \end{bmatrix}$$

Μόρφωση τοπικών μητρώων δυσκαμψίας (P2) λόγω εκκεντρότητας

$$\left[k^2\right] = \left[e^2\right]^T \left[k^2\right]\left[e^2\right]$$



$$\left[k^2\right] = EI \begin{array}{c} \begin{array}{cccccc} \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} \end{array} \\ \begin{bmatrix} 2.706 & 0 & 0 & -2.706 & 0 & 0 \\ 0 & 0.036 & 0.125 & 0 & -0.036 & 0.208 \\ 0 & 0.125 & 0.577 & 0 & -0.125 & 0.577 \\ -2.706 & 0 & 0 & 2.706 & 0 & 0 \\ 0 & -0.036 & -0.125 & 0 & 0.036 & -0.208 \\ 0 & 0.208 & 0.577 & 0 & -0.208 & 1.346 \end{bmatrix} \end{array} \begin{array}{c} \mathbf{4} \\ \mathbf{5} \\ \mathbf{6} \\ \mathbf{7} \\ \mathbf{8} \\ \mathbf{9} \end{array}$$

Μόρφωση μητρώων μετασχηματισμού (P2)

$$\left[\Lambda^{i}_{PPF}\right]=\begin{bmatrix} \cos\varphi_i & \sin\varphi_i & 0 & & & \\ -\sin\varphi_i & \cos\varphi_i & 0 & & [0] & \\ 0 & 0 & 1 & & & \\ & & & \cos\varphi_i & \sin\varphi_i & 0 \\ & [0] & & -\sin\varphi_i & \cos\varphi_i & 0 \\ & & & 0 & 0 & 1 \end{bmatrix} \quad (i=1,2,3)$$

$$\varphi_1 = 0^o$$

$$\underbrace{\varphi_{2\_\alpha\rho\chi\acute{\eta}} = 330^o \qquad \varphi_{2\_\pi\acute{\epsilon}\rho\alpha\varsigma} = 270^o}$$

$$\Downarrow$$

$$\left[\Lambda^{1}_{PPF}\right]=\left[I\right]$$

$$\left[\Lambda^{2}_{PPF}\right]=\left[\begin{array}{ccc|ccc} 0.866 & -0.5 & 0 & & & \\ 0.5 & 0.866 & 0 & & [0] & \\ 0 & 0 & 1 & & & \\ \hline & & & 0 & -1 & 0 \\ & [0] & & 1 & 0 & 0 \\ & & & 0 & 0 & 1 \end{array}\right]$$

Μόρφωση καθολικών μητρώων δυσκαμψίας: $\left[\bar{k}^i\right]=\left[\Lambda^i_{PPF}\right]^T\left[k^i\right]\left[\Lambda^i_{PPF}\right]$

$$\left[\bar{k}^1\right]=EI\begin{array}{cccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & \\ \hline \end{array}$$

$$\left[\bar{k}^1\right]=EI\left[\begin{array}{cccccc} 3.125 & 0 & 0 & -3.125 & 0 & 0 \\ 0 & 0.055 & 0.166 & 0 & -0.055 & 0.166 \\ 0 & 0.166 & 0.666 & 0 & -0.166 & 0.333 \\ -3.125 & 0 & 0 & 3.125 & 0 & 0 \\ 0 & -0.055 & -0.166 & 0 & 0.055 & -0.166 \\ 0 & 0.166 & 0.333 & 0 & -0.166 & 0.666 \end{array}\right]\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

$$\left[\bar{k}^1\right]=\left[\Lambda^1_{PPF}\right]^T\left[k^1\right]\left[\Lambda^1_{PPF}\right]=\left[k^1\right]$$

$$\left[\bar{k}^2\right]=\left[\Lambda^2_{PPF}\right]^T\left[k^2\right]\left[\Lambda^2_{PPF}\right]$$

$$\left[\bar{k}^2\right]=EI\left[\begin{array}{cccccc} 2.039 & -1.156 & 0.062 & -0.018 & 2.343 & 0.104 \\ -1.156 & 0.704 & 0.108 & -0.031 & -1.353 & 0.180 \\ 0.062 & 0.108 & 0.577 & -0.125 & 0 & 0.578 \\ -0.018 & -0.031 & -0.125 & 0.036 & 0 & -0.208 \\ 2.343 & -1.353 & 0 & 0 & 2.706 & 0 \\ 0.104 & 0.180 & 0.578 & -0.208 & 0 & 1.346 \end{array}\right]\begin{array}{c} 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array}$$

Μόρφωση ολικού μητρώου δυσκαμψίας του φορέα

$$
\begin{array}{ccccccccc}
\multicolumn{3}{c}{\overbrace{\hspace{4cm}}^{\#1}} & \multicolumn{3}{c}{\overbrace{\hspace{4cm}}^{\#2}} & \multicolumn{3}{c}{\overbrace{\hspace{4cm}}^{\#3}} \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9
\end{array}
$$

$$
[\bar{k}] = EI \left[\begin{array}{ccccccccc}
3.125 & & & & & & & & \\
0 & 0.055 & & & & & & & \\
0 & 0.166 & 0.666 & & & & & & \\
-3.125 & 0 & 0 & 5.164 & & & & & \\
0 & -0.055 & -0.166 & -1.156 & 0.759 & & & & \\
0 & 0.166 & 0.333 & 0.0625 & -0.058 & 1.243 & & & \\
0 & 0 & 0 & -0.018 & -0.031 & -0.125 & 0.928 & & \\
0 & 0 & 0 & 2.343 & -1.353 & 0 & 0 & 2.706 & \\
0 & 0 & 0 & 0.104 & 0.180 & 0.578 & -0.208 & 0 & 1.346
\end{array}\right]
\begin{array}{l}
\left.\begin{array}{c} 1 \\ 2 \\ 3 \end{array}\right\} \#1 \\
\left.\begin{array}{c} 4 \\ 5 \\ 6 \end{array}\right\} \#2 \\
\left.\begin{array}{c} 7 \\ 8 \\ 9 \end{array}\right\} \#3
\end{array}
$$

$$
όπου\ k'_{77} = k_{77} + \frac{k_s}{EI} = 0.036 + \frac{24 \cdot 10^4}{26.88 \cdot 10^4} = 0.928
$$

Αναδιάταξη ολικού μητρώου δυσκαμψίας του φορέα
γνωστές-άγνωστες ποσότητες

$$
[V] = \begin{array}{c c}
 & \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\
\begin{array}{c} 6 \\ 7 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 8 \\ 9 \end{array} &
\left[\begin{array}{ccccccccc}
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\hdashline
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\end{array}
$$

$$
\left[\bar{k}_m\right] = [V]\left[\bar{k}\right][V]^T
$$

$$
\Rightarrow
$$

Αναδιάταξη ολικού μητρώου δυσκαμψίας του φορέα γνωστές-άγνωστες ποσότητες

Μητρώο δυσκαμψίας φορέα

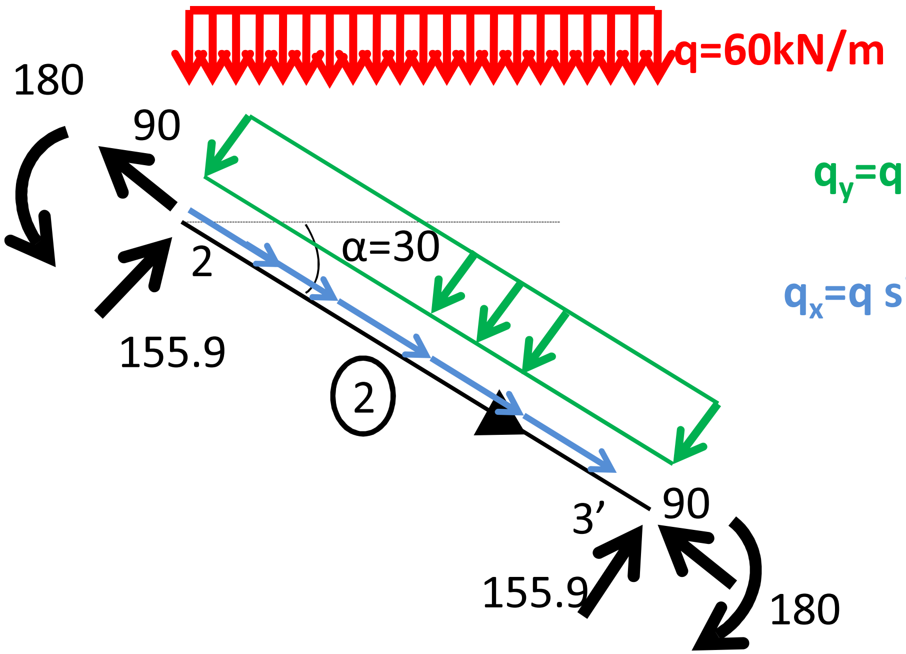
$\left[\bar{k}_{ff}\right]$ $\left[\bar{k}_{fs}\right]$

$$
\left[\bar{k}_m\right] = EI
\begin{array}{l}
\begin{array}{ccccccccc} \quad 6 & \quad\quad 7 & \quad\quad 1 & \quad\quad 2 & \quad\quad 3 & \quad\quad 4 & \quad\quad 5 & \quad\quad 8 & \quad\quad 9 \end{array} \\
\left[\begin{array}{cc|ccccccc}
1.243 & & & & & & & & \\
-0.125 & 0.928 & & & & & & & \\
\hline
0 & 0 & 3.125 & & & & & & \\
0.166 & 0 & 0 & 0.055 & & & & & \\
0.333 & 0 & 0 & 0.166 & 0.666 & & & & \\
0.0625 & -0.018 & -3.125 & 0 & 0 & 5.164 & & & \\
-0.058 & -0.031 & 0 & -0.055 & -0.166 & -1.156 & 0.759 & & \\
0 & 0 & 0 & 0 & 0 & 2.343 & -1.353 & 2.706 & \\
0.578 & -0.208 & 0 & 0 & 0 & 0.104 & 0.180 & 0 & 1.346
\end{array}\right]
\begin{array}{c} 6 \\ 7 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 8 \\ 9 \end{array}
\end{array}
$$

$\left[\bar{k}_{sf}\right]$ $\left[\bar{k}_{ss}\right]$

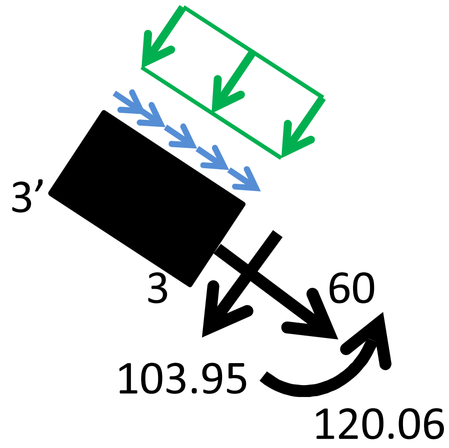
Υπολογισμός δράσεων παγιώσεως μελών:

$$\left\{\bar{A}_r^i\right\} = \left[\hat{\Lambda}_{PPT}^i\right]^T \left\{A_r^i\right\} = \begin{Bmatrix} \bar{A}_r^{ij} \\ \bar{A}_r^{ik} \end{Bmatrix}$$



q_y=q cos^2(a)=45kN/m

q_x=q sin(a) cos(a) =26kN/m



$$\left\{\bar{A}_r^1\right\} = \left[\Lambda_{PPF}^1\right]^T \left\{A_r^1\right\} = \{0\}$$

$$\left\{\bar{A}_r^2\right\} = \left[\Lambda_{PPF}^2\right]^T [e^2]^T \left\{A_r^2\right\} = \left[\Lambda_{PPF}^2\right]^T [e^2]^T \begin{Bmatrix} -90 \\ 155.9 \\ 180 \\ -90 \\ 155.9 \\ -180 \end{Bmatrix} = \begin{Bmatrix} 0.01 \\ 180 \\ 180 \\ 155.9 \\ 90 \\ -539.97 \end{Bmatrix}$$

Υπολογισμός δράσεων παγιώσεως κόμβων:

$$\left\{\bar{S}^1\right\} = [\bar{A}_r^{1j}] = \{0\}$$

$$\left\{\bar{S}^2\right\} = [\bar{A}_r^{1k}] + [\bar{A}_r^{2j}] = \begin{Bmatrix} 0.01 \\ 180 \\ 180 \end{Bmatrix}$$

$$\left\{\bar{S}^3\right\} = [\bar{A}_r^{2k}] = \begin{Bmatrix} 155.9 \\ 90 \\ -539.97 \end{Bmatrix}$$

Μόρφωση αναδιατεταγμένων μητρώων εξωτερικών δράσεων και μετακινήσεων κόμβων

$$\{\bar{P}_m\} = [V]\{\bar{P}\} = \begin{Bmatrix} \bar{P}_f \\ \bar{P}_s \end{Bmatrix} = \left\{ \begin{array}{c} -180 \\ -155.9-103.95 \\ \hdashline \bar{P}_1^1 \\ \bar{P}_2^1 \\ \bar{M}_3^1 \\ \bar{P}_1^2 - 0.01 \\ \bar{P}_2^2 - 180 \\ \bar{P}_2^3 - 90 + 60 \\ \bar{M}_3^3 + 539.97 + 120.06 \end{array} \right\} \begin{array}{c} 6 \\ 7 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 8 \\ 9 \end{array}$$

$$\{\bar{\Delta}_m\} = [V]\{\bar{\Delta}\} = \begin{Bmatrix} \bar{\Delta}_f \\ \bar{\Delta}_s \end{Bmatrix} = \left\{ \begin{array}{c} \bar{\theta}_3^2 = \Delta 6 \\ \bar{u}_1^3 = \Delta 7 \\ \hdashline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right\} \begin{array}{c} 6 \\ 7 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 8 \\ 9 \end{array}$$

Επίλυση συστήματος

$$\begin{Bmatrix} \bar{P}_f \\ \bar{P}_s \end{Bmatrix} = \begin{bmatrix} \bar{k}_{ff} & \bar{k}_{fs} \\ \bar{k}_{sf} & \bar{k}_{ss} \end{bmatrix} \begin{Bmatrix} \bar{\Delta}_f \\ \bar{\Delta}_s \end{Bmatrix}$$

Αποτελέσματα:

Υπολογισμός αγνώστων μετακινήσεων των κόμβων [Δ_f] και αγνώστων αντιδράσεων στις στηρίξεις [P_s]

$$\left\{\overline{\Delta}_f\right\} = \begin{Bmatrix} \overline{\theta}_3^2 = \Delta 6 \\ \overline{u}_1^3 = \Delta 7 \end{Bmatrix} = \begin{Bmatrix} -6.523 \\ -11.296 \end{Bmatrix} \cdot 10^{-4}$$

$$\left\{\overline{P}_s\right\} = \begin{Bmatrix} \overline{P}_1^1 \\ \overline{P}_2^1 \\ \overline{M}_3^1 \\ \overline{P}_1^2 \\ \overline{P}_2^2 \\ \overline{P}_2^3 \\ \overline{M}_3^3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -29.107 \\ -58.390 \\ -5.484 \\ 199.583 \\ 30 \\ -698.224 \end{Bmatrix}$$

$$\overline{P}_1^3 = -\Delta 7 \cdot k_s = 11.296 \cdot 24 = 271.104 kN$$

Υπολογισμός ακραίων δράσεων μελών

$$[A^i] = [A_r^i] + [k^i][\Lambda_{PPF}^i][\overline{D}^i]$$

Υπολογισμός ακραίων δράσεων μελών

$$[A^1]=[0]+[k^1][I]\begin{Bmatrix}0\\0\\0\\0\\0\\-6.523\end{Bmatrix}\cdot 10^{-4}=\begin{Bmatrix}0\\-29.107\\-58.390\\0\\29.107\\-116.775\end{Bmatrix}$$

Μέλος (1)

Μέλος (2)

$$[A^2]=[A_r^2]+[k^2][\Lambda_{PPF}^2][e^2]\begin{Bmatrix}0\\0\\-6.23\\-11.296\\0\\0\end{Bmatrix}=\begin{Bmatrix}-90\\144.913\\116.789\\-90\\166.886\\-192.542\end{Bmatrix}$$

Διαγράμματα εντατικών μεγεθών [M],[Q] και [N]

Διαγράμματα εντατικών μεγεθών [M],[Q] και [N]