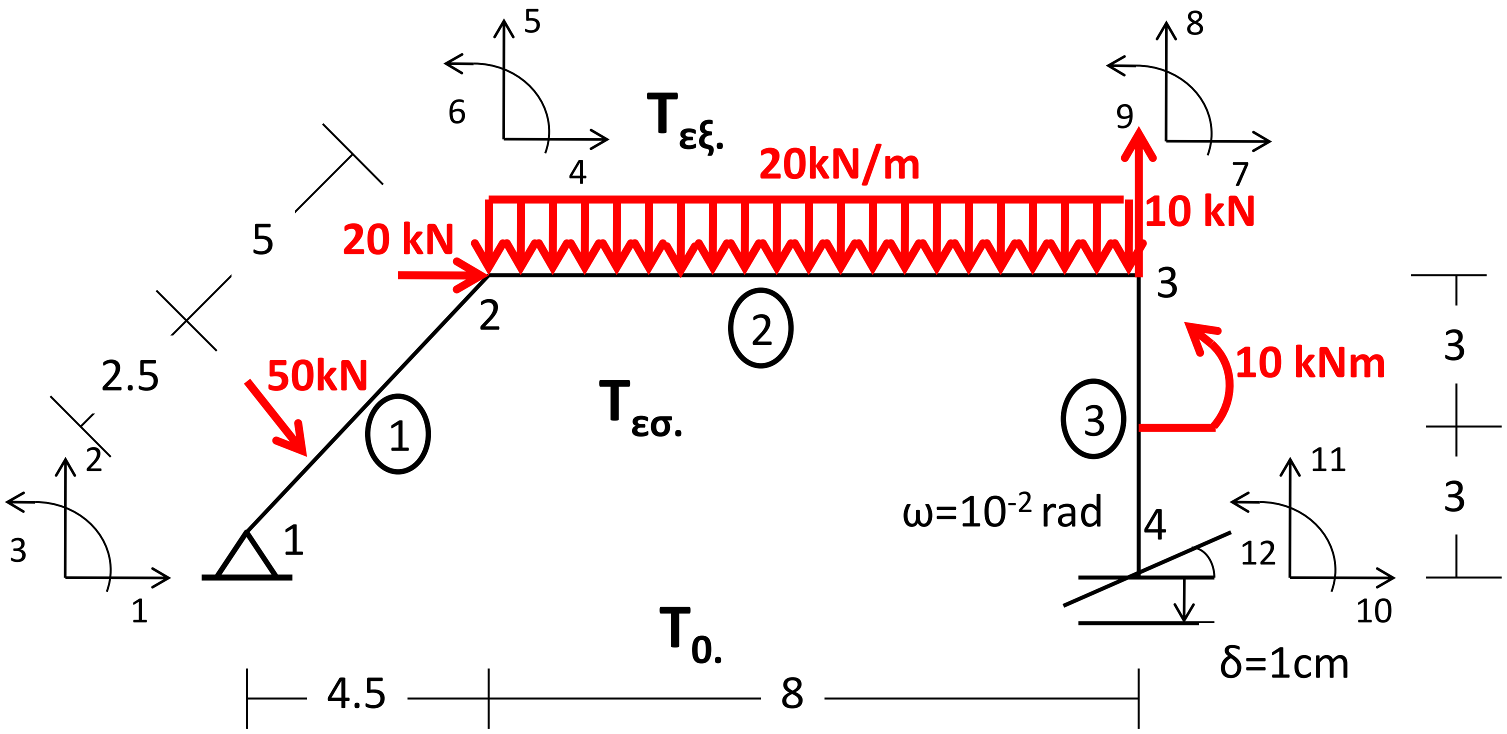
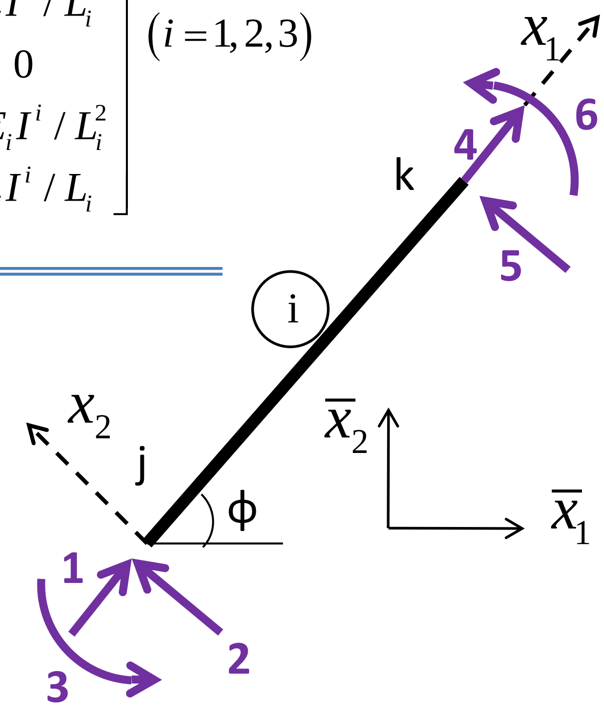
Άσκηση Πλαισίου



Μόρφωση τοπικών μητρώων δυσκαμψίας (P2)

$$\left[k^i\right]=\begin{bmatrix} A_iE_i/L_i & 0 & 0 & -A_iE_i/L_i & 0 & 0 \\ 0 & 12E_iI^i/L_i^3 & 6E_iI^i/L_i^2 & 0 & -12E_iI^i/L_i^3 & 6E_iI^i/L_i^2 \\ 0 & 6E_iI^i/L_i^2 & 4E_iI^i/L_i & 0 & -6E_iI^i/L_i^2 & 2E_iI^i/L_i \\ -A_iE_i/L_i & 0 & 0 & A_iE_i/L_i & 0 & 0 \\ 0 & -12E_iI^i/L_i^3 & -6E_iI^i/L_i^2 & 0 & 12E_iI^i/L_i^3 & -6E_iI^i/L_i^2 \\ 0 & 6E_iI^i/L_i^2 & 2E_iI^i/L_i & 0 & -6E_iI^i/L_i^2 & 4E_iI^i/L_i \end{bmatrix} \quad (i=1,2,3)$$



$$\left[k^1\right]=EI\begin{array}{c} \begin{matrix} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \end{matrix} \\ \begin{bmatrix} 10 & 0 & 0 & -10 & 0 & 0 \\ 0 & 0.0284 & 0.1067 & 0 & -0.0284 & 0.1067 \\ 0 & 0.1067 & 0.5333 & 0 & -0.1067 & 0.2667 \\ -10 & 0 & 0 & 10 & 0 & 0 \\ 0 & -0.0284 & -0.1067 & 0 & 0.0284 & -0.1067 \\ 0 & 0.1067 & 0.2667 & 0 & -0.1067 & 0.5333 \end{bmatrix} \end{array}$$

$$\left[k^2\right]=EI\begin{array}{c} \begin{matrix} \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} \end{matrix} \\ \begin{bmatrix} 9.375 & 0 & 0 & -9.375 & 0 & 0 \\ 0 & 0.0234 & 0.0938 & 0 & -0.0234 & 0.0938 \\ 0 & 0.0938 & 0.5 & 0 & -0.0938 & 0.25 \\ -9.375 & 0 & 0 & 9.375 & 0 & 0 \\ 0 & -0.0234 & -0.0938 & 0 & 0.0234 & -0.0938 \\ 0 & 0.0938 & 0.25 & 0 & -0.0938 & 0.5 \end{bmatrix} \end{array}$$

$$\left[k^3\right]=EI\begin{array}{c} \begin{matrix} \mathbf{7} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} \end{matrix} \\ \begin{bmatrix} 12.5 & 0 & 0 & -12.5 & 0 & 0 \\ 0 & 0.0556 & 0.1670 & 0 & -0.0556 & 0.1670 \\ 0 & 0.1670 & 0.6667 & 0 & -0.1670 & 0.3333 \\ -12.5 & 0 & 0 & 12.5 & 0 & 0 \\ 0 & -0.0556 & -0.1670 & 0 & 0.0556 & -0.1670 \\ 0 & 0.1670 & 0.3333 & 0 & -0.1670 & 0.6667 \end{bmatrix} \end{array}$$

Μόρφωση μητρώων μετασχηματισμού (P2)

$$\left[\Lambda_{PPF}^{i}\right]=\begin{bmatrix} \cos\varphi_i & \sin\varphi_i & 0 & & & \\ -\sin\varphi_i & \cos\varphi_i & 0 & & [0] & \\ 0 & 0 & 1 & & & \\ & & & \cos\varphi_i & \sin\varphi_i & 0 \\ & [0] & & -\sin\varphi_i & \cos\varphi_i & 0 \\ & & & 0 & 0 & 1 \end{bmatrix} \left(i=1,2,3\right)$$

$$\left[\Lambda_{PPF}^{1}\right]=\begin{bmatrix} 0.6 & 0.8 & 0 & & & \\ -0.8 & 0.6 & 0 & & [0] & \\ 0 & 0 & 1 & & & \\ & & & 0.6 & 0.8 & 0 \\ & [0] & & -0.8 & 0.6 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$

$$\left[\Lambda_{PPF}^{2}\right]=\left[I\right]=\begin{bmatrix} 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & [0] & \\ 0 & 0 & 1 & & & \\ & & & 1 & 0 & 0 \\ & [0] & & 0 & 1 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$

$$\left[\Lambda_{PPF}^{3}\right]=\begin{bmatrix} 0 & -1 & 0 & & & \\ 1 & 0 & 0 & & [0] & \\ 0 & 0 & 1 & & & \\ & & & 0 & -1 & 0 \\ & [0] & & 1 & 0 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$

Μόρφωση καθολικών μητρώων δυσκαμψίας: $\left[\bar{k}^{i}\right]=\left[\Lambda_{PPF}^{i}\right]^{T}\left[k^{i}\right]\left[\Lambda_{PPF}^{i}\right]$

$$\left[\bar{k}^{i}\right]=\left[\begin{array}{ccc|cccc}
\bar{k}_{11}^{i} & & & & & & \\
\bar{k}_{21}^{i} & \bar{k}_{22}^{i} & & & & & \\
\bar{k}_{31}^{i} & \bar{k}_{32}^{i} & \bar{k}_{33}^{i} & & & & \\
\hline
\bar{k}_{41}^{i} & \bar{k}_{42}^{i} & \bar{k}_{43}^{i} & & \bar{k}_{44}^{i} & & \\
\bar{k}_{51}^{i} & \bar{k}_{52}^{i} & \bar{k}_{53}^{i} & & \bar{k}_{54}^{i} & \bar{k}_{55}^{i} & \\
\bar{k}_{61}^{i} & \bar{k}_{62}^{i} & \bar{k}_{63}^{i} & & \bar{k}_{64}^{i} & \bar{k}_{65}^{i} & \bar{k}_{66}^{i}
\end{array}\right]$$

$$\left(i=1,2,3\right)$$

$$\left[\bar{k}^{1}\right]=EI\begin{array}{c}
\begin{array}{cccccc} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \end{array} \\
\begin{bmatrix}
3.618 & & & & & \\
4.786 & 6.410 & & & & \\
-0.085 & 0.064 & 0.5333 & & & \\
-3.618 & -4.786 & 0.085 & 3.618 & & \\
4.786 & -6.410 & -0.064 & 4.786 & 6.410 & \\
-0.085 & 0.064 & 0.2667 & 0.085 & -0.064 & 0.5333
\end{bmatrix}
\end{array}$$

$$\left[\bar{k}^{2}\right]=EI\begin{array}{c}
\begin{array}{cccccc} \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} \end{array} \\
\begin{bmatrix}
9.375 & & & & & \\
0 & 0.0234 & & & & \\
0 & 0.0938 & 0.5 & & & \\
-9.375 & 0 & 0 & 9.375 & & \\
0 & -0.0234 & -0.0938 & 0 & 0.0234 & \\
0 & 0.0938 & 0.25 & 0 & -0.0938 & 0.5
\end{bmatrix}
\end{array}$$

$$\left[\bar{k}^{3}\right]=EI\begin{array}{c}
\begin{array}{cccccc} \mathbf{7} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} \end{array} \\
\begin{bmatrix}
0.0556 & & & & & \\
0 & 12.5 & & & & \\
0.1670 & 0 & 0.6667 & & & \\
-0.0556 & 0 & -0.1670 & 0.0556 & & \\
0 & -12.5 & 0 & 0 & 12.5 & \\
0.1670 & 0 & 0.3333 & -0.1670 & 0 & 0.6667
\end{bmatrix}
\end{array}$$

Μόρφωση ολικού μητρώου δυσκαμψίας του φορέα

$$
\left[\bar{k}\right] = \begin{array}{c} \\ \textbf{\#1} \\ \textbf{\#2} \\ \textbf{\#3} \\ \textbf{\#4} \end{array}
\begin{array}{c}
\begin{array}{cccc} \quad\textbf{\#1} & \qquad\qquad\textbf{\#2} & \qquad\qquad\textbf{\#3} & \qquad\qquad\textbf{\#4} \end{array} \\
\begin{bmatrix}
\bar{k}_{jj}^{1} & \bar{k}_{jk}^{1} & [0] & [0] \\
\bar{k}_{kj}^{1} & \bar{k}_{kk}^{1} + \bar{k}_{jj}^{2} & \bar{k}_{jk}^{2} & [0] \\
[0] & \bar{k}_{kj}^{2} & \bar{k}_{kk}^{2} + \bar{k}_{jj}^{3} & \bar{k}_{jk}^{3} \\
[0] & [0] & \bar{k}_{kj}^{3} & \bar{k}_{kk}^{3}
\end{bmatrix}
\end{array}
$$

Μόρφωση ολικού μητρώου δυσκαμψίας του φορέα

$$
\left[\bar{k}\right] = EI
\begin{array}{cc|cccccccccccc|}
 & & \multicolumn{3}{c}{\#1} & \multicolumn{3}{c}{\#2} & \multicolumn{3}{c}{\#3} & \multicolumn{3}{c}{\#4} \\
 & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
\hline
 & 1 & 3.618 & & & & & & & & & & & \\
\#1 & 2 & 4.786 & 6.410 & & & & & & & & & & \\
 & 3 & -0.085 & 0.064 & 0.533 & & & & & & & & & \\
 & 4 & -3.618 & -4.786 & 0.085 & 12.993 & & & & & & & & \\
\#2 & 5 & -4.786 & -6.410 & -0.064 & 4.786 & 6.433 & & & & & & & \\
 & 6 & -0.085 & 0.064 & 0.266 & 0.085 & 0.030 & 1.033 & & & & & & \\
 & 7 & 0 & 0 & 0 & -9.375 & 0 & 0 & 9.430 & & & & & \\
\#3 & 8 & 0 & 0 & 0 & 0 & -0.023 & -0.094 & 0 & 12.523 & & & & \\
 & 9 & 0 & 0 & 0 & 0 & 0.094 & 0.25 & 0.167 & -0.094 & 1.167 & & & \\
 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & -0.055 & 0 & -0.167 & 0.055 & & \\
\#4 & 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -12.5 & 0 & 0 & 12.5 & \\
 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0.167 & 0 & 0.333 & -0.167 & 0 & 0.667 \\
\hline
\end{array}
$$

Αναδιάταξη ολικού μητρώου δυσκαμψίας του φορέα γνωστές-άγνωστες ποσότητες

$$[V] = \begin{array}{cc} & \begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array} \\ \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ \\ 1 \\ 2 \\ 10 \\ 11 \\ 12 \end{array} & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ - & - & - & - & - & - & - & - & - & - & - & - \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\left[\bar{k}_m\right] = [V]\left[\bar{k}\right][V]^T$$

Αναδιάταξη ολικού μητρώου δυσκαμψίας του φορέα γνωστές-άγνωστες ποσότητες

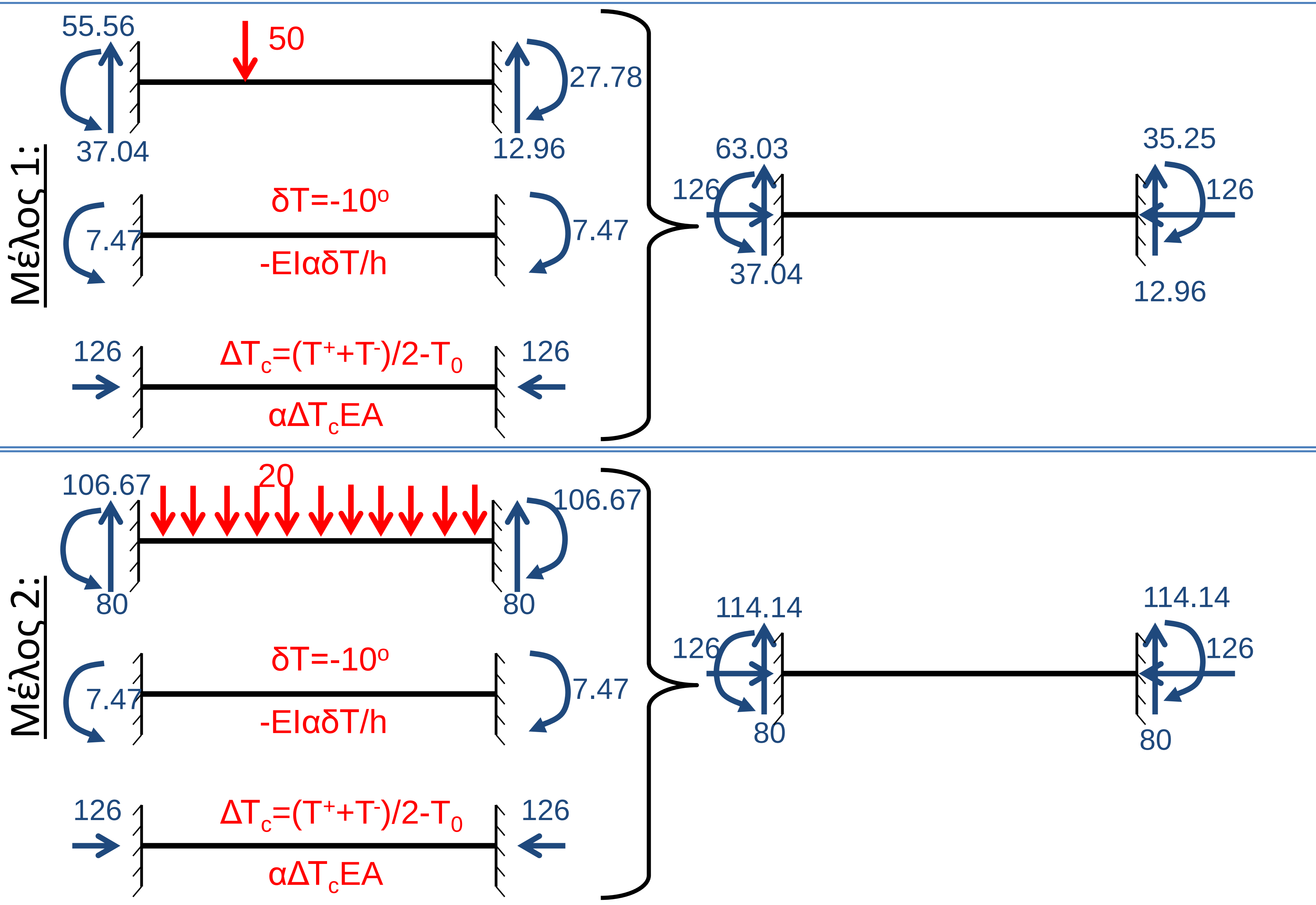
Μητρώο δυσκαμψίας φορέα

$$\left[\bar{k}_m\right] = EI \left[\begin{array}{ccccccc|ccccc}
0.533 & & & & & & & & & & & \\
0.085 & 12.993 & & & & & & & & & & \\
-0.064 & 4.786 & 6.433 & & & & & & & & & \\
0.266 & 0.085 & 0.030 & 1.033 & & & & & & & & \\
0 & -9.375 & 0 & 0 & 9.430 & & & & & & & \\
0 & 0 & -0.023 & -0.094 & 0 & 12.523 & & & & & & \\
0 & 0 & 0.094 & 0.25 & 0.167 & -0.094 & 0.167 & & & & & \\
\hline
-0.085 & -3.618 & -4.786 & -0.085 & 0 & 0 & 0 & 3.618 & & & & \\
0.064 & -4.786 & -6.410 & 0.064 & 0 & 0 & 0 & 4.786 & 6.410 & & & \\
0 & 0 & 0 & 0 & -0.055 & 0 & -0.167 & 0 & 0 & 0.055 & & \\
0 & 0 & 0 & 0 & 0 & -12.5 & 0 & 0 & 0 & 0 & 12.5 & \\
0 & 0 & 0 & 0 & 0.167 & 0 & 0.333 & 0 & 0 & -0.667 & 0 & 0.667
\end{array}\right]$$

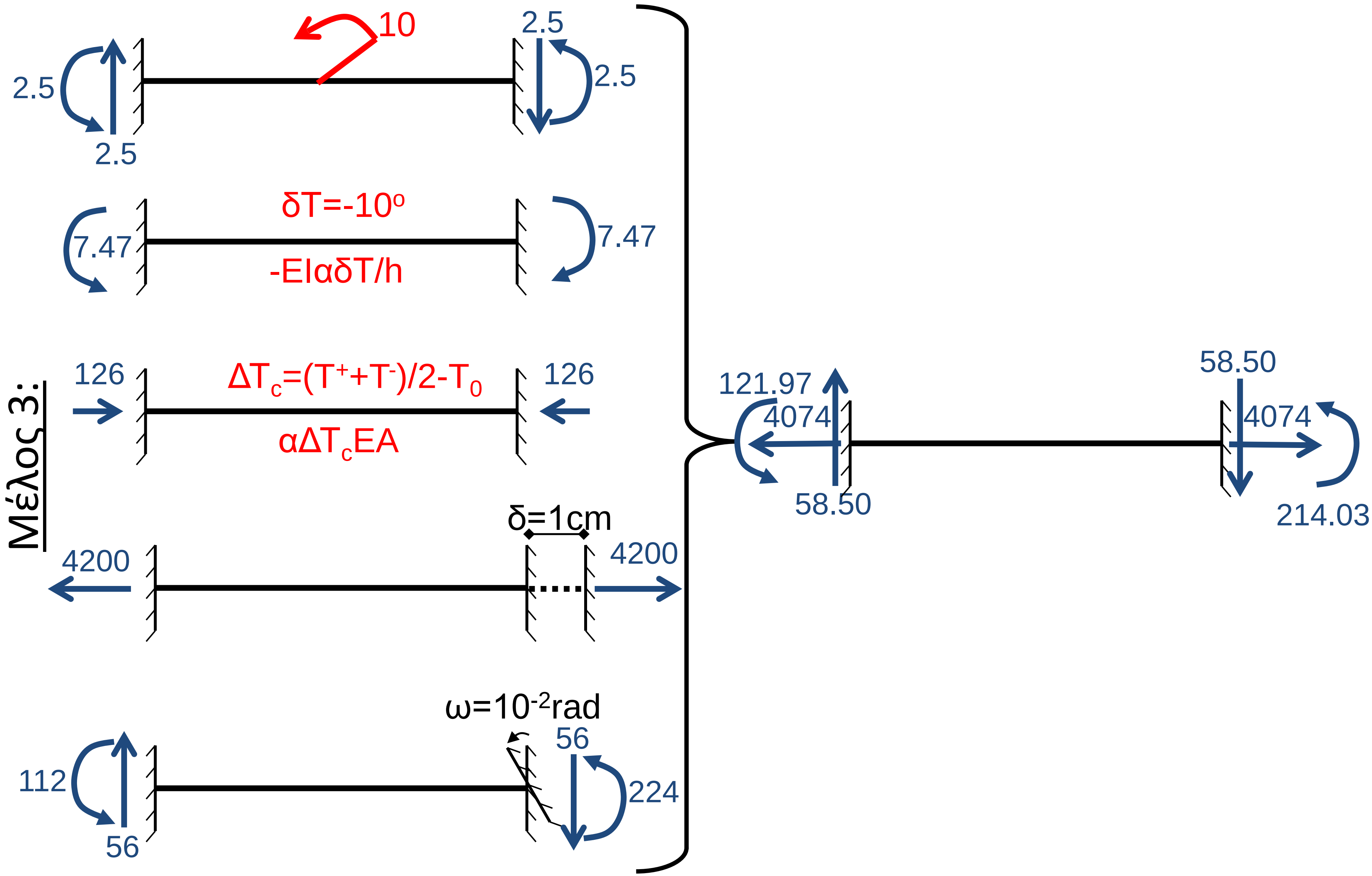
$\left[\bar{k}\right]$ $\left[\bar{k}_{fs}\right]$

$\left[\bar{k}_{sf}\right]$ $\left[\bar{k}_{ss}\right]$

Υπολογισμός δράσεων παγιώσεως μελών:



Υπολογισμός δράσεων παγιώσεως μελών:



Υπολογισμός δράσεων παγιώσεως μελών:

$$\left\{\bar{A}_r^i\right\} = \left[\hat{\Lambda}_{PPF}^i\right]^T \left\{A_r^i\right\} = \begin{Bmatrix} \bar{A}_r^{ij} \\ \bar{A}_r^{ik} \end{Bmatrix}$$

$$\left\{\bar{A}_r^1\right\} = \left[\Lambda_{PPF}^1\right]^T \left\{A_r^1\right\} = \begin{Bmatrix} 45.968 \\ 123.024 \\ 63.03 \\ -85.968 \\ -93.024 \\ -35.25 \end{Bmatrix} \quad \left\{\bar{A}_r^2\right\} = \left[\Lambda_{PPF}^2\right]^T \left\{A_r^2\right\} = \begin{Bmatrix} 126 \\ 80 \\ 114.14 \\ -126 \\ 80 \\ -114.14 \end{Bmatrix} \quad \left\{\bar{A}_r^3\right\} = \left[\Lambda_{PPF}^3\right]^T \left\{A_r^3\right\} = \begin{Bmatrix} 58.5 \\ 4074 \\ 121.97 \\ -58.5 \\ -4074 \\ 219.03 \end{Bmatrix}$$

Υπολογισμός δράσεων παγιώσεως κόμβων:

$$\left\{\bar{S}^1\right\} = \left\{\bar{A}_r^{1j}\right\} = \begin{Bmatrix} 45.968 \\ 123.024 \\ 63.03 \end{Bmatrix} \qquad \left\{\bar{S}^2\right\} = \left\{\bar{A}_r^{1k}\right\} + \left\{\bar{A}_r^{2j}\right\} = \begin{Bmatrix} 40.032 \\ -13.024 \\ 78.890 \end{Bmatrix}$$

$$\left\{\bar{S}^3\right\} = \left\{\bar{A}_r^{2k}\right\} + \left\{\bar{A}_r^{3j}\right\} = \begin{Bmatrix} -67.50 \\ 4154 \\ 7.83 \end{Bmatrix} \qquad \left\{\bar{S}^4\right\} = \left\{\bar{A}_r^{3k}\right\} = \begin{Bmatrix} -58.5 \\ -4074 \\ 219.03 \end{Bmatrix}$$

Μόρφωση αναδιατεταγμένων μητρώων εξωτερικών δράσεων και μετακινήσεων κόμβων

$$\left\{\bar{P}_m\right\}=\left\{\begin{matrix}\bar{P}\\ \bar{P}_s\end{matrix}\right\}=\left\{\begin{matrix}-63.03\\ 20-40\\ 13.024\\ -78.89\\ 67.5\\ 10-4154\\ -7.83\\ --------\\ \bar{P}_1^1-45.968\\ \bar{P}_2^1-123.024\\ \bar{P}_1^4+58.5\\ \bar{P}_2^4+4074\\ \bar{M}_3^4-219.03\end{matrix}\right\}$$

$$\left\{\bar{\Delta}\right\}=\left\{\begin{matrix}\bar{\Delta}\\ \bar{\Delta}_s\end{matrix}\right\}=\left\{\begin{matrix}\bar{\theta}_3^1\\ \bar{u}_1^2\\ \bar{u}_2^2\\ \bar{\theta}_3^2\\ \bar{u}_1^3\\ \bar{u}_2^3\\ \bar{\theta}_3^3\\ --\\ 0\\ 0\\ 0\\ 0\\ 0\end{matrix}\right\}$$

Επίλυση συστήματος

$$\left\{\begin{matrix}\bar{P}\\ \bar{P}_s\end{matrix}\right\}=\begin{bmatrix}\bar{k} & \bar{k}_{fs}\\ \bar{k}_{sf} & \bar{k}_{ss}\end{bmatrix}\left\{\begin{matrix}\bar{\Delta}\\ \bar{\Delta}_s\end{matrix}\right\}$$

Αποτελέσματα:

Υπολογισμός αγνώστων μετατοπίσεων των κόμβων [Δ] και αγνώστων αντιδράσεων στις στηρίξεις [P_s]

$$\left\{\bar{\Delta}\right\} = \begin{Bmatrix} \bar{\theta}_3^1 \\ \bar{u}_1^2 \\ \bar{u}_2^2 \\ \bar{\theta}_3^2 \\ \bar{u}_1^3 \\ \bar{u}_2^3 \\ \bar{\theta}_3^3 \end{Bmatrix} = \frac{1}{EI} \begin{Bmatrix} -298.453 \\ 851.778 \\ -634.191 \\ -58.077 \\ 855.589 \\ -333.18 \\ -82.066 \end{Bmatrix}$$

$$\left\{\bar{P}_s\right\} = \begin{Bmatrix} \bar{P}_1^1 \\ \bar{P}_2^1 \\ \bar{P}_1^4 \\ \bar{P}_2^4 \\ \bar{M}_3^4 \end{Bmatrix} = \begin{Bmatrix} 30.276 \\ 89.116 \\ -90.274 \\ 90.875 \\ 330.403 \end{Bmatrix}$$

Υπολογισμός ακραίων δράσεων μελών

$$[A^i] = [A_r^i] + [k^i][\Lambda_{PPF}^i][\bar{D}^i]$$

Υπολογισμός ακραίων δράσεων μελών

$$\begin{Bmatrix} F_1^{1j} \\ F_2^{1j} \\ M_3^{1j} \\ F_1^{1k} \\ F_2^{1k} \\ M_3^{1k} \end{Bmatrix} = \begin{Bmatrix} 88.86 \\ 28.979 \\ 1.062 \\ -88.86 \\ 21.021 \\ -33.021 \end{Bmatrix} (\approx 0.0)$$

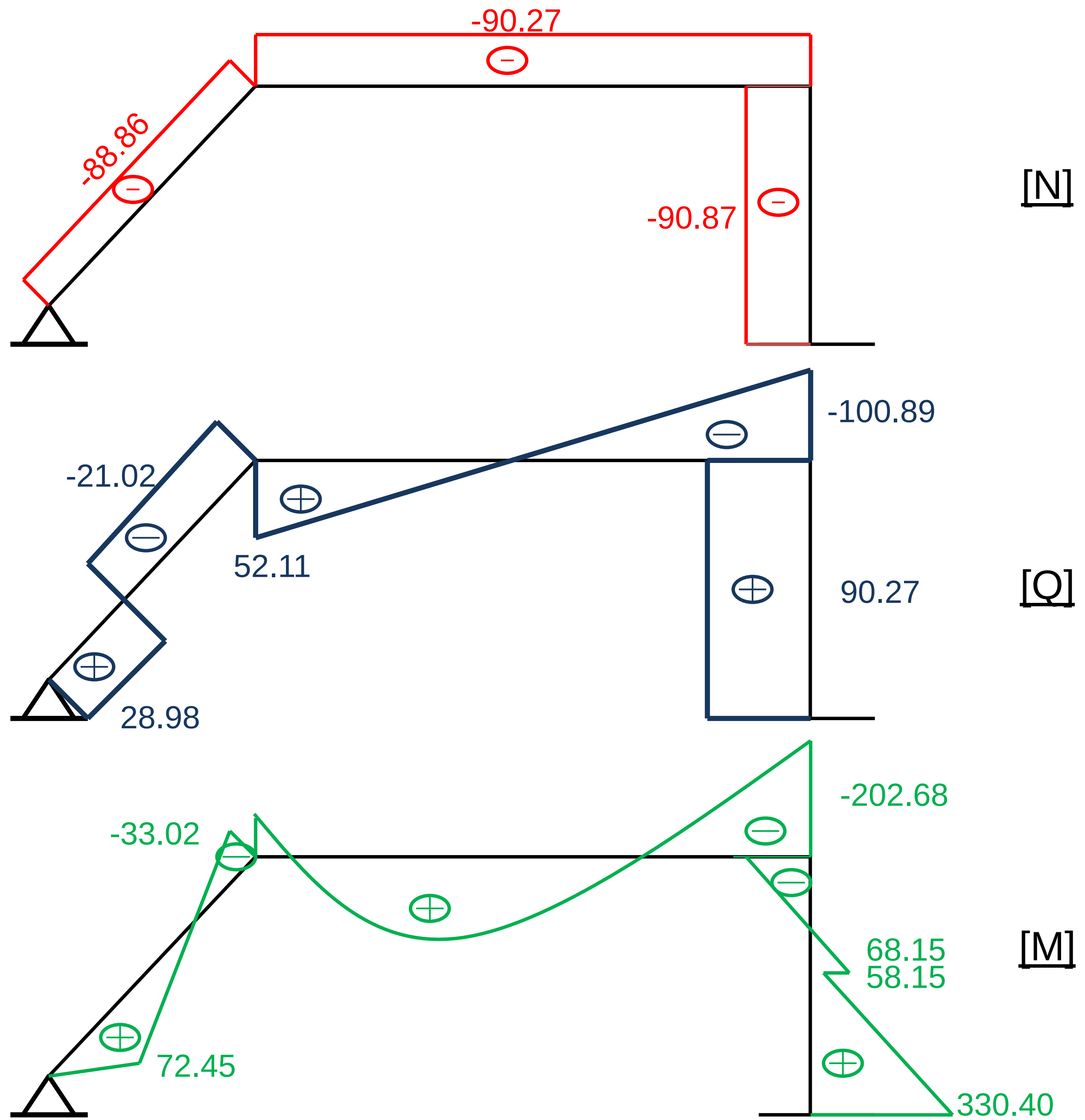
$$\begin{Bmatrix} F_1^{2j} \\ F_2^{2j} \\ M_3^{2j} \\ F_1^{2k} \\ F_2^{2k} \\ M_3^{2k} \end{Bmatrix} = \begin{Bmatrix} 90.272 \\ 59.114 \\ 34.084 \\ -90.272 \\ 100.886 \\ -202.677 \end{Bmatrix} (\approx 33.021)$$

$$\begin{Bmatrix} F_1^{3j} \\ F_2^{3j} \\ M_3^{3j} \\ F_1^{3k} \\ F_2^{3k} \\ M_3^{3k} \end{Bmatrix} = \begin{Bmatrix} 90.875 \\ 90.274 \\ 202.677 \\ -90.875 \\ -90.274 \\ 330.403 \end{Bmatrix}$$

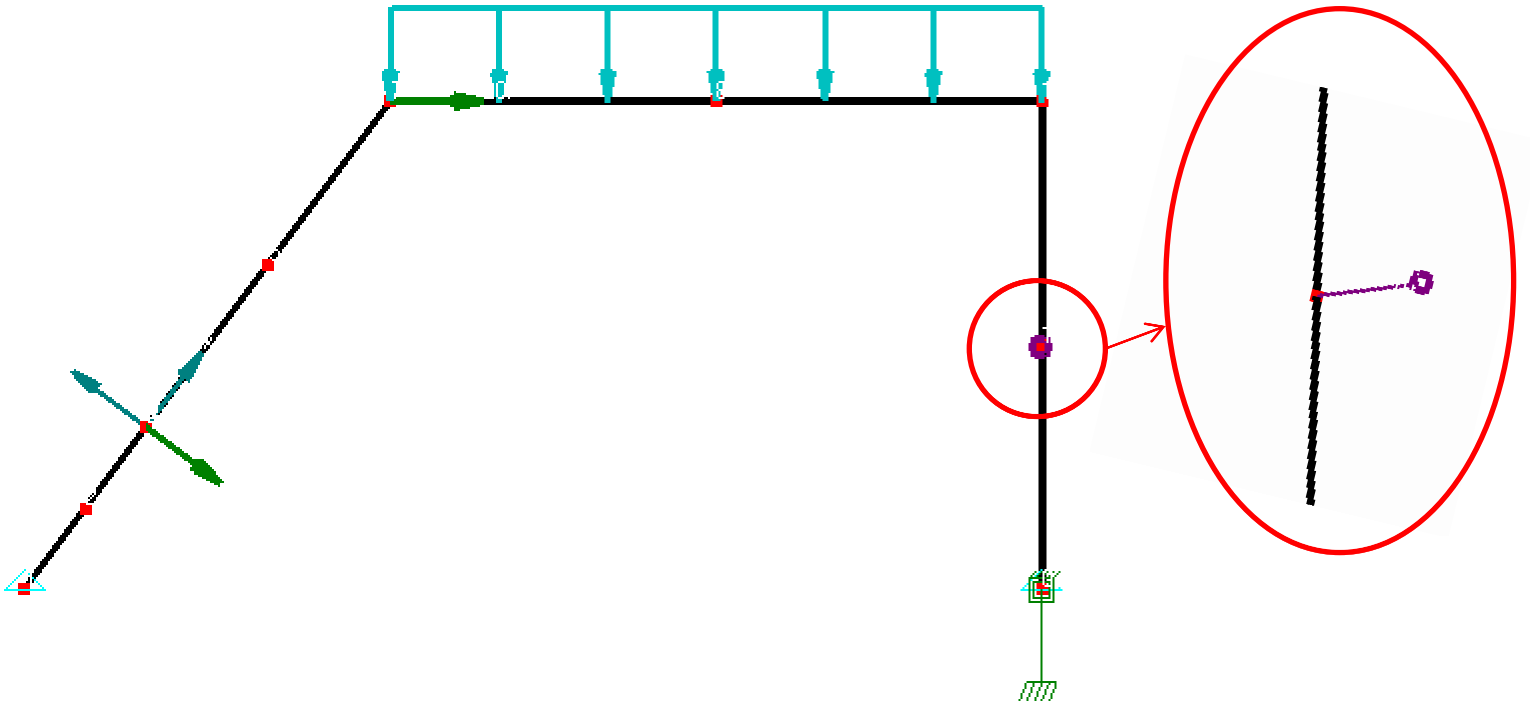
$\Rightarrow$ Διαγράμματα εντατικών μεγεθών [M],[Q] και [N]

$\Downarrow$

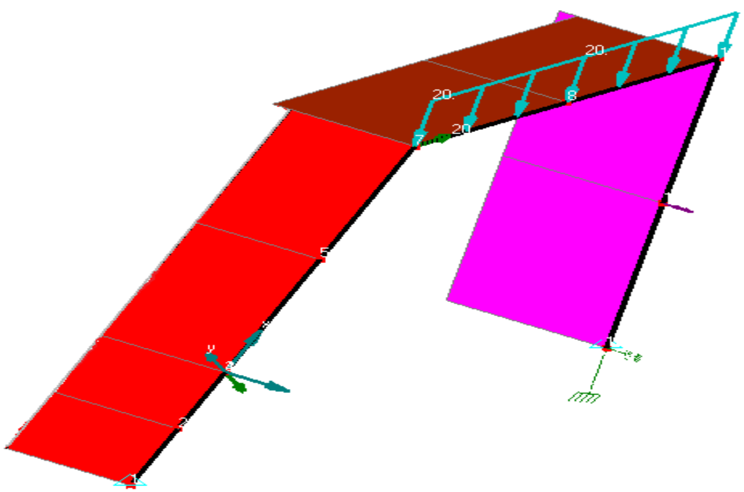
Διαγράμματα εντατικών μεγεθών [M],[Q] και [N]



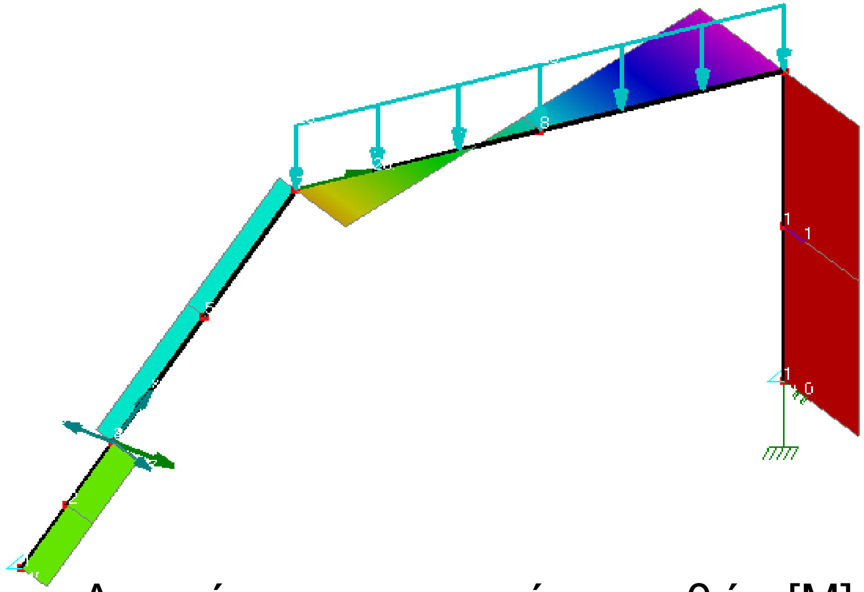
Υπολογισμός με NASTRAN



Διαγράμματα εντατικών μεγεθών [N]



Διαγράμματα εντατικών μεγεθών [Q]



Διαγράμματα εντατικών μεγεθών [M]