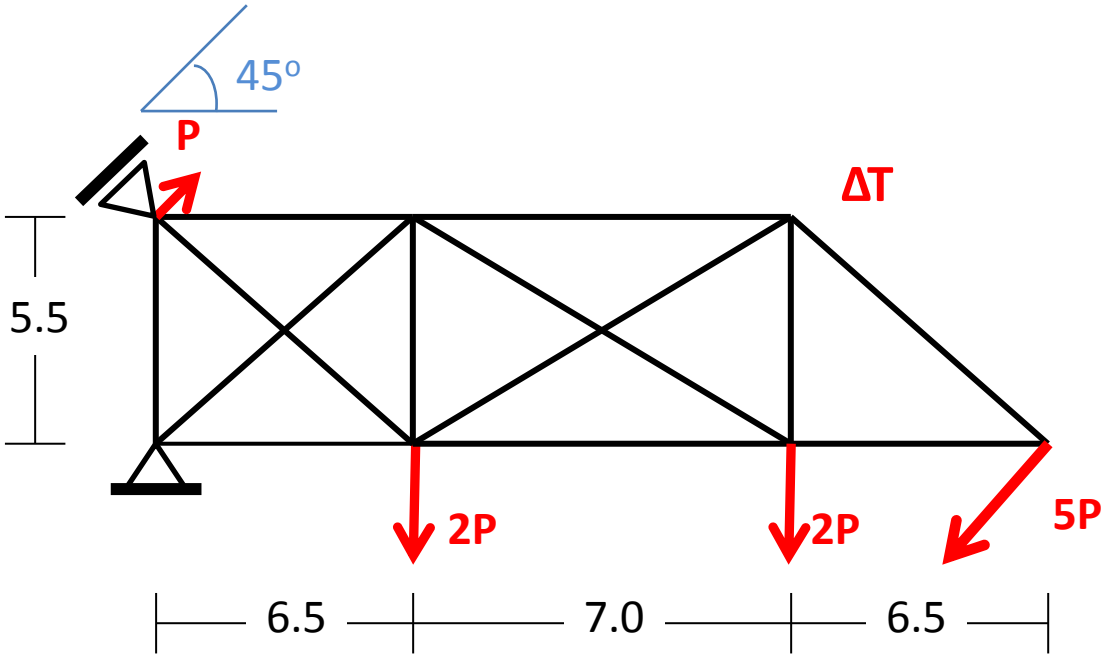
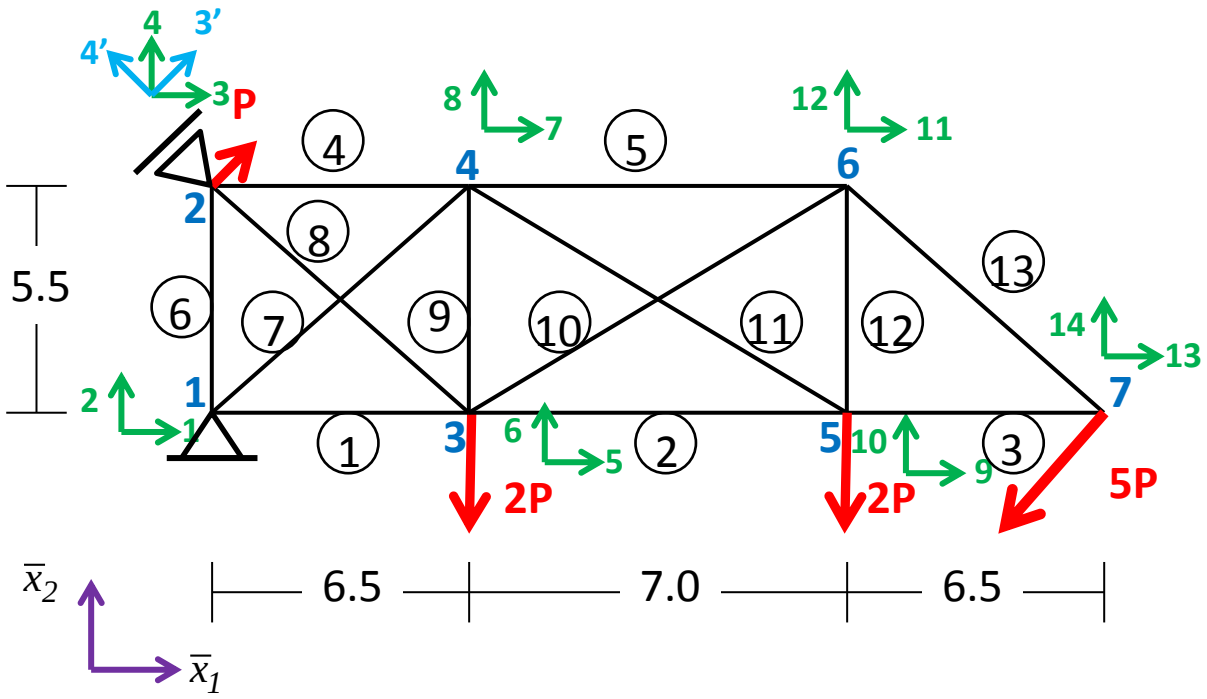
Άσκηση Δικτυώματος



Αρίθμηση κόμβων, μελών και βαθμών ελευθερίας



E= 2.1*10^{8}kN/m^2 A= 12cm^2

α=10^{-5}

P= 50kN

ΔT= T-T$_0$ = 35^o-15^o =20^o

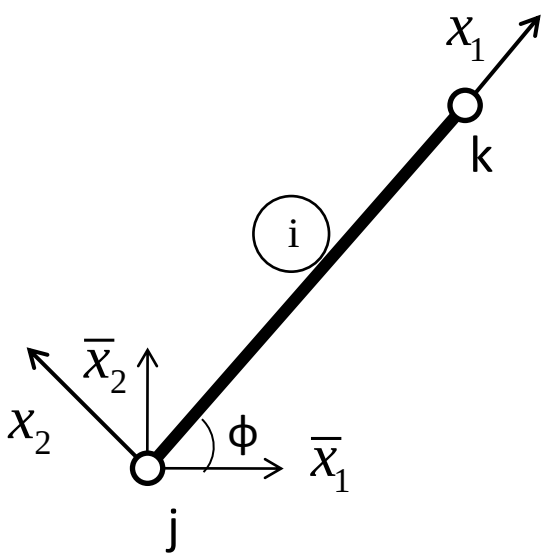
1, 2, 3, 4, 5, 13 : 2A

6, 9, 12 : 1.5A

7, 8, 10, 11 : A

Μόρφωση τοπικών μητρώων δυσκαμψίας (P1)

$$[\hat{k}^i] = \frac{A_i E_i}{L_i}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$



$$[\hat{k}^1] = [\hat{k}^3] = [\hat{k}^4] = \begin{bmatrix} 77538.4615 & -77538.4615 \\ -77538.4615 & 77538.4615 \end{bmatrix}$$

$$[\hat{k}^2] = [\hat{k}^5] = \begin{bmatrix} 72000.000 & -72000.000 \\ -72000.000 & 72000.000 \end{bmatrix}$$

$$[\hat{k}^6] = [\hat{k}^9] = [\hat{k}^{12}] = \begin{bmatrix} 68727.273 & -68727.273 \\ -68727.273 & 68727.273 \end{bmatrix}$$

$$[\hat{k}^7] = [\hat{k}^8] = \begin{bmatrix} 29595.899 & -29595.899 \\ -29595.899 & 29595.899 \end{bmatrix}$$

$$[\hat{k}^{10}] = [\hat{k}^{11}] = \begin{bmatrix} 28307.460 & -28307.460 \\ -28307.460 & 28307.460 \end{bmatrix}$$

$$[\hat{k}^{13}] = \begin{bmatrix} 59191.798 & -59191.798 \\ -59191.798 & 59191.798 \end{bmatrix}$$

Μόρφωση μητρώων μετασχηματισμού (P1)

$$\left[\hat{\Lambda}^{i}_{PPT}\right]=\begin{bmatrix}\cos\varphi^{i} & \sin\varphi^{i} & 0 & 0\\ 0 & 0 & \cos\varphi^{i} & \sin\varphi^{i}\end{bmatrix}$$

$$\left[\hat{\Lambda}^{1}_{PPT}\right]=\left[\hat{\Lambda}^{2}_{PPT}\right]=\left[\hat{\Lambda}^{3}_{PPT}\right]=\left[\hat{\Lambda}^{4}_{PPT}\right]=\left[\hat{\Lambda}^{5}_{PPT}\right]=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

$$\left[\hat{\Lambda}^{6}_{PPT}\right]=\left[\hat{\Lambda}^{9}_{PPT}\right]=\left[\hat{\Lambda}^{12}_{PPT}\right]=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$\left[\hat{\Lambda}^{7}_{PPT}\right]=\begin{bmatrix}0.763386 & 0.645942 & 0 & 0\\ 0 & 0 & 0.763386 & 0.645942\end{bmatrix}$$

$$\left[\hat{\Lambda}^{8}_{PPT}\right]=\left[\hat{\Lambda}^{13}_{PPT}\right]=\begin{bmatrix}0.763386 & -0.645942 & 0 & 0\\ 0 & 0 & 0.763386 & -0.645942\end{bmatrix}$$

$$\left[\hat{\Lambda}^{10}_{PPT}\right]=\begin{bmatrix}0.786318 & 0.617822 & 0 & 0\\ 0 & 0 & 0.786318 & 0.617822\end{bmatrix}$$

$$\left[\hat{\Lambda}^{11}_{PPT}\right]=\begin{bmatrix}0.786318 & -0.617822 & 0 & 0\\ 0 & 0 & 0.786318 & -0.617822\end{bmatrix}$$

Μόρφωση καθολικών μητρώων δυσκαμψίας: $\left[\bar{k}^{i}\right]=\left[\hat{\Lambda}^{i}_{PPT}\right]^{T}\left[\hat{k}^{i}\right]\left[\hat{\Lambda}^{i}_{PPT}\right]$

$$\begin{matrix} & 1 & 2 & 5 & 6 \end{matrix}$$
$$\left[\bar{k}^{1}\right]=\begin{bmatrix} 77538.46 & 0 & -77538.46 & 0 \\ 0 & 0 & 0 & 0 \\ -77538.46 & 0 & 77538.46 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} & 3 & 4 & 7 & 8 \end{matrix}$$
$$\left[\bar{k}^{4}\right]=\begin{bmatrix} 77538.46 & 0 & -77538.46 & 0 \\ 0 & 0 & 0 & 0 \\ -77538.46 & 0 & 77538.46 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} & 5 & 6 & 9 & 10 \end{matrix}$$
$$\left[\bar{k}^{2}\right]=\begin{bmatrix} 72000.00 & 0 & -72000.00 & 0 \\ 0 & 0 & 0 & 0 \\ -72000.00 & 0 & 72000.00 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} & 7 & 8 & 11 & 12 \end{matrix}$$
$$\left[\bar{k}^{5}\right]=\begin{bmatrix} 72000.00 & 0 & -72000.00 & 0 \\ 0 & 0 & 0 & 0 \\ -72000.00 & 0 & 72000.00 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} & 9 & 10 & 13 & 14 \end{matrix}$$
$$\left[\bar{k}^{3}\right]=\begin{bmatrix} 77538.46 & 0 & -77538.46 & 0 \\ 0 & 0 & 0 & 0 \\ -77538.46 & 0 & 77538.46 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} & 1 & 2 & 3 & 4 \end{matrix}$$
$$\left[\bar{k}^{6}\right]=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 68727.27 & 0 & -68727.27 \\ 0 & 0 & 0 & 0 \\ 0 & -68727.27 & 0 & 68727.27 \end{bmatrix}$$

$$\left[\bar{k}^{7}\right]=\begin{array}{c}\begin{matrix}1 & 2 & 7 & 8\end{matrix}\\ \begin{bmatrix}17247.27 & 14593.84 & -17247.27 & -14593.84\\ 14593.84 & 12348.63 & -14593.84 & -12348.63\\ -17247.27 & -14593.84 & 17247.27 & 14593.84\\ -14593.84 & -12348.63 & 14593.84 & 12348.63\end{bmatrix}\end{array}$$

$$\left[\bar{k}^{10}\right]=\begin{array}{c}\begin{matrix}5 & 6 & 11 & 12\end{matrix}\\ \begin{bmatrix}17502.40 & 13751.89 & -17502.40 & -13751.89\\ 13751.89 & 10805.06 & -13751.89 & -10805.06\\ -17502.40 & -13751.89 & 17502.40 & 13751.89\\ -13751.89 & -10805.06 & 13751.89 & 10805.06\end{bmatrix}\end{array}$$

$$\left[\bar{k}^{8}\right]=\begin{array}{c}\begin{matrix}3 & 4 & 5 & 6\end{matrix}\\ \begin{bmatrix}17247.27 & -14593.84 & -17247.27 & 14593.84\\ -14593.84 & 12348.63 & 14593.84 & -12348.63\\ -17247.27 & 14593.84 & 17247.27 & -14593.84\\ 14593.84 & -12348.63 & -14593.84 & 12348.63\end{bmatrix}\end{array}$$

$$\left[\bar{k}^{11}\right]=\begin{array}{c}\begin{matrix}7 & 8 & 9 & 10\end{matrix}\\ \begin{bmatrix}17502.40 & -13751.89 & -17502.40 & 13751.89\\ -13751.89 & 10805.06 & 13751.89 & -10805.06\\ -17502.40 & 13751.89 & 17502.40 & -13751.89\\ 13751.89 & -10805.06 & -13751.89 & 10805.06\end{bmatrix}\end{array}$$

$$\left[\bar{k}^{9}\right]=\begin{array}{c}\begin{matrix}5 & 6 & 7 & 8\end{matrix}\\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 68727.27 & 0 & -68727.27\\ 0 & 0 & 0 & 0\\ 0 & -68727.27 & 0 & 68727.27\end{bmatrix}\end{array}$$

$$\left[\bar{k}^{12}\right]=\begin{array}{c}\begin{matrix}9 & 10 & 11 & 12\end{matrix}\\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 68727.27 & 0 & -68727.27\\ 0 & 0 & 0 & 0\\ 0 & -68727.27 & 0 & 68727.27\end{bmatrix}\end{array}$$

$$\left[\bar{k}^{13}\right]=\begin{array}{c}\begin{matrix}11 & 12 & 13 & 14\end{matrix}\\ \begin{bmatrix}34494.53 & -29187.68 & -34494.53 & 29187.68\\ -29187.68 & 24697.27 & 29187.68 & -24697.27\\ -34494.53 & 29187.68 & 34494.53 & -29187.68\\ 29187.68 & -24697.27 & -29187.68 & 24697.27\end{bmatrix}\end{array}$$

Μόρφωση ολικού μητρώου δυσκαμψίας του φορέα

$$
\left[\bar{k}\right] =
\begin{array}{c}
\\
\#1 \\
\#2 \\
\#3 \\
\#4 \\
\#5 \\
\#6 \\
\#7
\end{array}
\begin{bmatrix}
\begin{array}{c}\#1\\ \bar{k}^{1}_{jj}+\bar{k}^{6}_{jj}\\ +\bar{k}^{7}_{jj}\end{array} & \begin{array}{c}\#2\\ \bar{k}^{6}_{jk}\end{array} & \begin{array}{c}\#3\\ \bar{k}^{1}_{jk}\end{array} & \begin{array}{c}\#4\\ \bar{k}^{7}_{jk}\end{array} & \begin{array}{c}\#5\\ [0]\end{array} & \begin{array}{c}\#6\\ [0]\end{array} & \begin{array}{c}\#7\\ [0]\end{array} \\
\bar{k}^{6}_{kj} & \bar{k}^{4}_{jj}+\bar{k}^{6}_{kk}+\bar{k}^{8}_{jj} & \bar{k}^{8}_{jk} & \bar{k}^{4}_{jk} & [0] & [0] & [0] \\
\bar{k}^{1}_{kj} & \bar{k}^{8}_{kj} & \bar{k}^{1}_{kk}+\bar{k}^{2}_{jj}+\bar{k}^{8}_{kk}+\bar{k}^{9}_{jj}+\bar{k}^{10}_{jj} & \bar{k}^{9}_{jk} & \bar{k}^{2}_{jk} & \bar{k}^{10}_{jk} & [0] \\
\bar{k}^{7}_{kj} & \bar{k}^{4}_{kj} & \bar{k}^{9}_{kj} & \bar{k}^{4}_{kk}+\bar{k}^{5}_{jj}+\bar{k}^{7}_{kk}+\bar{k}^{9}_{kk}+\bar{k}^{11}_{jj} & \bar{k}^{11}_{jk} & \bar{k}^{5}_{jk} & [0] \\
[0] & [0] & \bar{k}^{2}_{kj} & \bar{k}^{11}_{kj} & \bar{k}^{2}_{kk}+\bar{k}^{3}_{jj}+\bar{k}^{11}_{kk}+\bar{k}^{12}_{jj} & \bar{k}^{12}_{jk} & \bar{k}^{3}_{jk} \\
[0] & [0] & \bar{k}^{10}_{kj} & \bar{k}^{5}_{kj} & \bar{k}^{12}_{kj} & \bar{k}^{5}_{kk}+\bar{k}^{10}_{kk}+\bar{k}^{12}_{kk}+\bar{k}^{13}_{jj} & \bar{k}^{13}_{jk} \\
[0] & [0] & [0] & [0] & \bar{k}^{3}_{kj} & \bar{k}^{13}_{kj} & \bar{k}^{3}_{kk}+\bar{k}^{13}_{kk}
\end{bmatrix}
$$

Μόρφωση ολικού μητρώου δυσκαμψίας του φορέα

$$[\bar{k}] = \begin{bmatrix}
94785.7 & & & & & & & & & & & & \\
14593.8 & 81075.9 & & & & & & & & & & & \\
0 & 0 & 94785.7 & & & & & & & & & & \\
0 & -68727.3 & -14593.8 & 81075.9 & & & & & & & & & \\
-77538.5 & 0 & -17247.3 & 14593.8 & 184288.1 & & & & & & & & \\
0 & 0 & 14593.8 & -12348.6 & -842.0 & 91881.0 & & & & & & & \\
-17247.3 & -14593.8 & -77538.5 & 0 & 0 & 0 & 184288.1 & & & & & & \\
-14593.8 & -12348.6 & 0 & 0 & 0 & -68727.3 & 842.0 & 91881.0 & & & & & \\
0 & 0 & 0 & 0 & -72000.0 & 0 & -17502.4 & 13751.9 & 167040.9 & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 13751.9 & -10805.1 & -13751.9 & 79532.3 & & & \\
0 & 0 & 0 & 0 & -17502.4 & -13751.9 & -72000.0 & 0 & 0 & 0 & 123996.9 & & \\
0 & 0 & 0 & 0 & -13751.9 & -10805.1 & 0 & 0 & 0 & -68727.3 & -15435.8 & 104229.6 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -77538.5 & 0 & -34494.5 & 29187.7 & 112033.0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 29187.7 & -24697.3 & -29187.7 & 24697.3
\end{bmatrix}$$

$$[R] = \begin{bmatrix}
\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbf{\cos\theta} & \mathbf{\sin\theta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbf{-\sin\theta} & \mathbf{\cos\theta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1}
\end{bmatrix}$$

Τροποποίηση ολικού μητρώου δυσκαμψίας του φορέα - συνθήκες στηρίξεως

$$\left[\bar{k}_m\right] = [R]\left[\bar{k}\right][R]^T \Longrightarrow$$

$$\left[\bar{k}_m\right] = \begin{bmatrix}
94785.7 & & & & & & & & & & & & \\
14593.8 & 81075.9 & & & & & & & & & & & \\
0 & 0 & 73337.0 & & & & & & & & & & \\
0 & 48597.5 & -6854.9 & 102524.7 & & & & & & & & & \\
-77538.5 & 0 & 1876.3 & -22515.1 & 184288.1 & & & & & & & & \\
0 & 0 & -1587.6 & 19051.2 & -842.0 & 91881.0 & & & & & & & \\
-17247.3 & -14593.8 & 54828.0 & -54828.0 & 0 & 0 & 184288.1 & & & & & & \\
-14593.8 & -12348.6 & 0 & 0 & 0 & -68727.3 & 842.0 & 91881.0 & & & & & \\
0 & 0 & 0 & 0 & -72000.0 & 0 & -17502.4 & 13751.9 & 167040.9 & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 13751.9 & -10805.1 & -13751.9 & 79532.3 & & & \\
0 & 0 & 0 & 0 & -17502.4 & -13751.9 & -72000.0 & 0 & 0 & 0 & 123996.9 & & \\
0 & 0 & 0 & 0 & -13751.9 & -10805.1 & 0 & 0 & 0 & -68727.3 & -15435.8 & 104229.6 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -77538.5 & 0 & -34494.5 & 29187.7 & 112033.0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 29187.7 & -24697.3 & -29187.7 & 24697.3
\end{bmatrix}$$

$$[V] = \begin{bmatrix}
0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} \\
\multicolumn{14}{c}{- -} \\
\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Αναδιάταξη ολικού μητρώου δυσκαμψίας του φορέα – γνωστές-άγνωστες ποσότητες

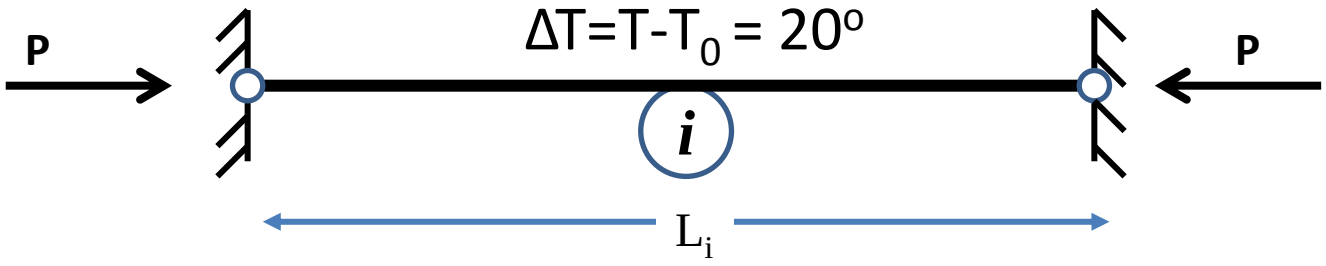
$$\left[\bar{k}_{mm}\right] = [V]\left[\bar{k}_m\right][V]^T$$

Μητρώο δυσκαμψίας φορέα

$$\left[\bar{k}_{mm}\right] = \begin{bmatrix}
73337.0 & & & & & & & & & & & & & \\
1876.3 & 184288.1 & & & & & & & & & & & & \\
-1587.6 & -842.0 & 91881.0 & & & & & & & & & & & \\
54828.0 & 0 & 0 & 184288.1 & & & & & & & & & & \\
0 & 0 & -68727.3 & 842.0 & 91881.0 & & & & & & & & & \\
0 & -72000.0 & 0 & -17502.4 & 13751.9 & 167040.9 & & & & & & & & \\
0 & 0 & 0 & 13751.9 & -10805.1 & -13751.9 & 79532.3 & & & & & & & \\
0 & -17502.4 & -13751.9 & -72000.0 & 0 & 0 & 0 & 123996.9 & & & & & & \\
0 & -13751.9 & -10805.1 & 0 & 0 & 0 & -68727.3 & -15435.8 & 104229.6 & & & & & \\
0 & 0 & 0 & 0 & 0 & -77538.5 & 0 & -34494.5 & 29187.7 & 112033.0 & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 29187.7 & -24697.3 & -29187.7 & 24697.3 & & & \\
0 & -77538.5 & 0 & -17247.3 & -14593.8 & 0 & 0 & 0 & 0 & 0 & 0 & 94785.7 & & \\
0 & 0 & 0 & -14593.8 & -12348.6 & 0 & 0 & 0 & 0 & 0 & 0 & 14593.8 & 81075.9 & \\
-6854.9 & -22515.1 & 19051.2 & -54828.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 48597.5 & 102524.7
\end{bmatrix}$$

Υπολογισμός δράσεων παγιώσεως μελών

Εξετάζουμε τον παγιωμένο φορέα και τοποθετούμε τις αντιδράσεις των αμφίαρθρωτών στοιχείων



$$\left\{A_r^i\right\} = \begin{Bmatrix} F_r^{ij} \\ F_r^{ik} \end{Bmatrix} = \begin{Bmatrix} \alpha \cdot \Delta T \cdot E^i \cdot A^i \\ -\alpha \cdot \Delta T \cdot E^i \cdot A^i \end{Bmatrix}$$

Καθολικό σύστημα

$$\left\{\bar{A}_r^i\right\} = \left[\hat{\Lambda}_{PPT}^i\right]^T \left\{A_r^i\right\} = \begin{Bmatrix} \bar{A}_r^{ij} \\ \bar{A}_r^{ik} \end{Bmatrix}$$

Υπολογισμός δράσεων παγιώσεως μελών:

$$\left\{\bar{A}_r^i\right\} = \left[\hat{\Lambda}_{PPT}^{\,i}\right]^T \left\{A_r^i\right\} = \begin{Bmatrix} \bar{A}_r^{ij} \\ \bar{A}_r^{ik} \end{Bmatrix}$$

$$\left\{\bar{A}_r^1\right\} = \left[\hat{\Lambda}_{PPT}^{\,1}\right]^T \left\{A_r^1\right\} = \begin{Bmatrix} 100.8 \\ 0 \\ -100.8 \\ 0 \end{Bmatrix}$$

$$\left\{\bar{A}_r^5\right\} = \left[\hat{\Lambda}_{PPT}^{\,5}\right]^T \left\{A_r^5\right\} = \begin{Bmatrix} 100.8 \\ 0 \\ -100.8 \\ 0 \end{Bmatrix}$$

$$\left\{\bar{A}_r^{10}\right\} = \left[\hat{\Lambda}_{PPT}^{\,10}\right]^T \left\{A_r^{10}\right\} = \begin{Bmatrix} 39.630 \\ 31.138 \\ -39.630 \\ -31.138 \end{Bmatrix}$$

$$\left\{\bar{A}_r^2\right\} = \left[\hat{\Lambda}_{PPT}^{\,2}\right]^T \left\{A_r^2\right\} = \begin{Bmatrix} 100.8 \\ 0 \\ -100.8 \\ 0 \end{Bmatrix}$$

$$\left\{\bar{A}_r^{6,9,12}\right\} = \left[\hat{\Lambda}_{PPT}^{\,6,9,12}\right]^T \left\{A_r^{6,9,12}\right\} = \begin{Bmatrix} 0 \\ 75.6 \\ 0 \\ -75.6 \end{Bmatrix}$$

$$\left\{\bar{A}_r^{11}\right\} = \left[\hat{\Lambda}_{PPT}^{\,11}\right]^T \left\{A_r^{11}\right\} = \begin{Bmatrix} 39.630 \\ -31.138 \\ -39.630 \\ 31.138 \end{Bmatrix}$$

$$\left\{\bar{A}_r^3\right\} = \left[\hat{\Lambda}_{PPT}^{\,3}\right]^T \left\{A_r^3\right\} = \begin{Bmatrix} 100.8 \\ 0 \\ -100.8 \\ 0 \end{Bmatrix}$$

$$\left\{\bar{A}_r^7\right\} = \left[\hat{\Lambda}_{PPT}^{\,7}\right]^T \left\{A_r^7\right\} = \begin{Bmatrix} 38.475 \\ 32.555 \\ -38.475 \\ 32.555 \end{Bmatrix}$$

$$\left\{\bar{A}_r^4\right\} = \left[\hat{\Lambda}_{PPT}^{\,4}\right]^T \left\{A_r^4\right\} = \begin{Bmatrix} 100.8 \\ 0 \\ -100.8 \\ 0 \end{Bmatrix}$$

$$\left\{\bar{A}_r^8\right\} = \left[\hat{\Lambda}_{PPT}^{\,8}\right]^T \left\{A_r^8\right\} = \begin{Bmatrix} 38.475 \\ -32.555 \\ -38.475 \\ 32.555 \end{Bmatrix}$$

$$\left\{\bar{A}_r^{13}\right\} = \left[\hat{\Lambda}_{PPT}^{\,13}\right]^T \left\{A_r^{13}\right\} = \begin{Bmatrix} 76.949 \\ -65.111 \\ -76.949 \\ 65.111 \end{Bmatrix}$$

Δράσεις παγιώσεως κόμβων: (από ισορροπία κόμβων)

Κόμβος #1 : $\left\{\bar{S}^{1}\right\}=\left\{\bar{A}_r^{1j}\right\}+\left\{\bar{A}_r^{6j}\right\}+\left\{\bar{A}_r^{7j}\right\}=\begin{Bmatrix}139.275\\108.155\end{Bmatrix}$ $\beta.\varepsilon.\left(3',4'\right)$

Κόμβος #2 : $\left\{\bar{S}^{2}\right\}=\left\{\bar{A}_r^{4j}\right\}+\left\{\bar{A}_r^{6k}\right\}+\left\{\bar{A}_r^{8j}\right\}=\begin{Bmatrix}139.275\\-108.155\end{Bmatrix}\Rightarrow\left\{\bar{S}_m^{2}\right\}=\begin{Bmatrix}174.96\\22.00\end{Bmatrix}$

Κόμβος #3 : $\left\{\bar{S}^{3}\right\}=\left\{\bar{A}_r^{1k}\right\}+\left\{\bar{A}_r^{2j}\right\}+\left\{\bar{A}_r^{8k}\right\}+\left\{\bar{A}_r^{9j}\right\}+\left\{\bar{A}_r^{10j}\right\}=\begin{Bmatrix}1.156\\139.294\end{Bmatrix}$

Κόμβος #4 : $\left\{\bar{S}^{4}\right\}=\left\{\bar{A}_r^{4k}\right\}+\left\{\bar{A}_r^{5j}\right\}+\left\{\bar{A}_r^{7k}\right\}+\left\{\bar{A}_r^{9k}\right\}+\left\{\bar{A}_r^{11j}\right\}=\begin{Bmatrix}1.156\\-139.294\end{Bmatrix}$

Κόμβος #5 : $\left\{\bar{S}^{5}\right\}=\left\{\bar{A}_r^{2k}\right\}+\left\{\bar{A}_r^{3j}\right\}+\left\{\bar{A}_r^{11k}\right\}+\left\{\bar{A}_r^{12j}\right\}=\begin{Bmatrix}-39.630\\106.738\end{Bmatrix}$

Κόμβος #6 : $\left\{\bar{S}^{6}\right\}=\left\{\bar{A}_r^{5k}\right\}+\left\{\bar{A}_r^{10k}\right\}+\left\{\bar{A}_r^{12k}\right\}+\left\{\bar{A}_r^{13j}\right\}=\begin{Bmatrix}-63.481\\-171.849\end{Bmatrix}$

Κόμβος #7 : $\left\{\bar{S}^{7}\right\}=\left\{\bar{A}_r^{3k}\right\}+\left\{\bar{A}_r^{13k}\right\}=\begin{Bmatrix}-177.749\\-65.111\end{Bmatrix}$

Μόρφωση αναδιατεταγμένων μητρώων εξωτερικών δράσεων και μετατοπίσεων κόμβων

$$\{\bar{P}_{mm}\} = \begin{Bmatrix} \bar{P} \\ \bar{P}_s \end{Bmatrix} = \begin{Bmatrix} 50 \\ -1.156 \\ -100 \\ -1.156 \\ 139.294 \\ 39.630 \\ -100 \\ 63.481 \\ 171.849 \\ -161.48 \\ -190.85 \\ \hline \bar{P}_1^1 \\ \bar{P}_2^1 \\ \bar{P}_{2'}^2 \end{Bmatrix} \begin{matrix} -174.959 \\ \\ -139.294 \\ \\ \\ \\ -106.738 \\ \\ +177.749 \\ -65.11 \\ \\ -139.275 \\ -108.155 \\ -22.00 \end{matrix}$$

$$\{\bar{P}_{mm}^{nodal} - \bar{S}_{mm}\}$$

$$\{\bar{\Delta}_{mm}\} = \begin{Bmatrix} \bar{\Delta} \\ \bar{\Delta}_s \end{Bmatrix} = \begin{Bmatrix} \bar{\Delta}_{1'}^2 \\ \bar{\Delta}_1^3 \\ \bar{\Delta}_2^3 \\ \bar{\Delta}_1^4 \\ \bar{\Delta}_2^4 \\ \bar{\Delta}_1^5 \\ \bar{\Delta}_2^5 \\ \bar{\Delta}_1^6 \\ \bar{\Delta}_2^6 \\ \bar{\Delta}_1^7 \\ \bar{\Delta}_2^7 \\ \hline 0 \\ 0 \\ 0 \end{Bmatrix}$$

Επίλυση συστήματος

$$\begin{Bmatrix} \bar{P} \\ \bar{P}_s \end{Bmatrix} = \begin{bmatrix} \bar{k} & \bar{k}_{fs} \\ \bar{k}_{sf} & \bar{k}_{ss} \end{bmatrix} \begin{Bmatrix} \bar{\Delta} \\ \bar{\Delta}_s \end{Bmatrix}$$

Αποτελέσματα

$$\{\bar{\Delta}\} = \begin{Bmatrix} \bar{\Delta}_{1'}^{2} \\ \bar{\Delta}_{1}^{3} \\ \bar{\Delta}_{2}^{3} \\ \bar{\Delta}_{1}^{4} \\ \bar{\Delta}_{2}^{4} \\ \bar{\Delta}_{1}^{5} \\ \bar{\Delta}_{2}^{5} \\ \bar{\Delta}_{1}^{6} \\ \bar{\Delta}_{2}^{6} \\ \bar{\Delta}_{1}^{7} \\ \bar{\Delta}_{2}^{7} \end{Bmatrix} = \begin{Bmatrix} 0.015496 \\ -0.01204 \\ -0.03785 \\ 0.022323 \\ -0.0364 \\ -0.01836 \\ -0.10332 \\ 0.029655 \\ -0.10269 \\ -0.02205 \\ -0.17416 \end{Bmatrix}$$

$\beta.\varepsilon.(3')$

$$\{\bar{P}_s\} = \begin{Bmatrix} \bar{P}_{1}^{1} \\ \bar{P}_{2}^{1} \\ \bar{P}_{2'}^{1} \end{Bmatrix} = \begin{Bmatrix} 1079.835 \\ -629.333 \\ 1567.711 \end{Bmatrix}$$

$$\begin{Bmatrix} \bar{\Delta}_{1}^{2} \\ \bar{\Delta}_{2}^{2} \end{Bmatrix} = \begin{Bmatrix} 0.01095 \\ 0.01095 \end{Bmatrix}$$

$\beta.\varepsilon.(3,4)$

Υπολογισμός ακραίων δράσεων μελών

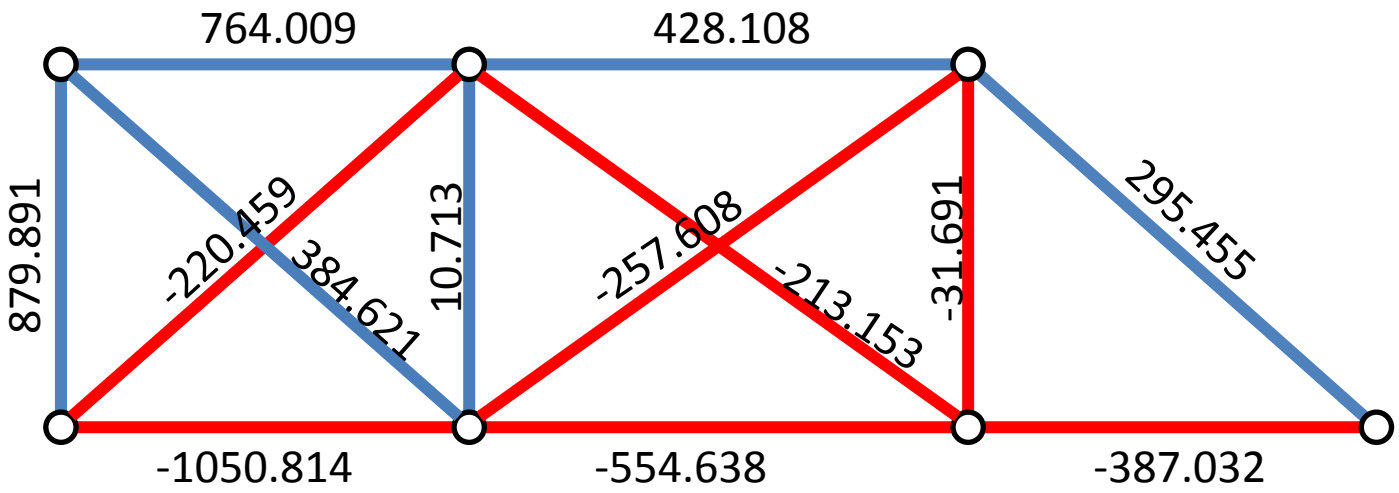
$$\left\{A^i\right\} = \left\{A_r^i\right\} + \left[k^i\right]\left[\Lambda^i\right]\left\{\bar{D}^i\right\}$$

$$\begin{Bmatrix} F_1^{1j} \\ F_1^{1k} \end{Bmatrix} = \begin{Bmatrix} 1050.814 \\ -1050.814 \end{Bmatrix} \qquad \begin{Bmatrix} F_1^{6j} \\ F_1^{6k} \end{Bmatrix} = \begin{Bmatrix} -879.891 \\ 879.891 \end{Bmatrix}$$

$$\begin{Bmatrix} F_1^{2j} \\ F_1^{2k} \end{Bmatrix} = \begin{Bmatrix} 554.638 \\ -554.638 \end{Bmatrix} \qquad \begin{Bmatrix} F_1^{7j} \\ F_1^{7k} \end{Bmatrix} = \begin{Bmatrix} 220.459 \\ -220.459 \end{Bmatrix} \qquad \begin{Bmatrix} F_1^{11j} \\ F_1^{11k} \end{Bmatrix} = \begin{Bmatrix} -213.153 \\ 213.153 \end{Bmatrix}$$

$$\begin{Bmatrix} F_1^{3j} \\ F_1^{3k} \end{Bmatrix} = \begin{Bmatrix} 387.032 \\ -387.032 \end{Bmatrix} \qquad \begin{Bmatrix} F_1^{8j} \\ F_1^{8k} \end{Bmatrix} = \begin{Bmatrix} -384.621 \\ 384.621 \end{Bmatrix} \qquad \begin{Bmatrix} F_1^{12j} \\ F_1^{12k} \end{Bmatrix} = \begin{Bmatrix} 31.691 \\ -31.691 \end{Bmatrix}$$

$$\begin{Bmatrix} F_1^{4j} \\ F_1^{4k} \end{Bmatrix} = \begin{Bmatrix} 764.0 \\ -764.0 \end{Bmatrix} \qquad \begin{Bmatrix} F_1^{9j} \\ F_1^{9k} \end{Bmatrix} = \begin{Bmatrix} -10.713 \\ 10.713 \end{Bmatrix} \qquad \begin{Bmatrix} F_1^{13j} \\ F_1^{13k} \end{Bmatrix} = \begin{Bmatrix} -295.455 \\ 295.455 \end{Bmatrix}$$

$$\begin{Bmatrix} F_1^{5j} \\ F_1^{5k} \end{Bmatrix} = \begin{Bmatrix} 428.108 \\ -428.108 \end{Bmatrix} \qquad \begin{Bmatrix} F_1^{10j} \\ F_1^{10k} \end{Bmatrix} = \begin{Bmatrix} 257.608 \\ -257.608 \end{Bmatrix}$$

Αξονικές δυνάμεις δικτυώματος