Ιούλιος 2011

Θέμα 1ο

α) $v(x,y) = x^3y - xy^3$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial v}{\partial y} = x^3 - 3xy^2 \qquad \frac{\partial v}{\partial x} = 3x^2y - y^3$$

$$\Rightarrow \frac{\partial u}{\partial x} = x^3 - 3xy^2 \ (1) \text{ και } \frac{\partial u}{\partial y} = y^3 - 3x^2y \ (2)$$

Ολοκληρώνουμε τη (2) ως προς y, άρα

$$u = y^4/4 - \frac{3}{2}x^2y^2 + a(x)$$

παραγωγίζουμε ως προς x, άρα

$$\frac{\partial u}{\partial x} = -3y^2x + a'(x) \quad (3)$$

$$(1)=(3) \Rightarrow x^3 - 3xy^2 = -3xy^2 + a'(x) \Rightarrow a'(x) = x^3 \Rightarrow \boxed{a(x) = \frac{x^4}{4}}$$

$$\Rightarrow u(x,y) = \frac{y^4}{4} - \frac{3}{2}x^2y^2 + \frac{x^4}{4}$$

β) $f(z) = \frac{1}{z-1}$, $z_0 = -i$

$$f'(z) = -(z-1)^{-2} \Rightarrow f'(z_0) = -(-i-1)^{-2} = -(i+1)^{-2}$$
$$f''(z) = 2(z-1)^{-3} \Rightarrow f''(z_0) = 2(-i-1)^{-3} = -2(i+1)^{-3}$$
$$f'''(z) = -6(z-1)^{-4} \Rightarrow f'''(z_0) = -6(-i-1)^{-4} = -6(i+1)^{-4}$$

$$\Rightarrow f(z) = -\frac{1}{i+1} - \frac{1}{(i+1)^2}\cdot(x+i) - \frac{2}{(i+1)^3}\frac{(x+i)^2}{2} - \frac{6(i+1)^{-4}}{6}(x+i)^3 - \dots$$

$$\Rightarrow f(z) = -\frac{1}{i+1} - \frac{x+i}{(i+1)^2} - \frac{(x+i)^2}{(i+1)^3} - \frac{(x+i)^3}{(i+1)^4} - \dots$$

$$\Rightarrow f(z) = \sum_{n=0}^{+\infty}\left(-\frac{(x+i)^n}{(i+1)^{n+1}}\right)$$