

Θέμα 2°

$$b) f(z) = \frac{x+a-iy}{x^2+y^2+2x+1} = (x+a) \cdot (x^2+y^2+2x+1)^{-1} - i y (x^2+y^2+2x+1)^{-1}$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{και} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{1}{x^2+y^2+2x+1} + (x+a)(-1)(x^2+y^2+2x+1)^{-2}(2x+2) =$$

$$= \frac{1}{x^2+y^2+2x+1} - 2(x+a)(x+1) \cdot \frac{1}{(x^2+y^2+2x+1)^2}$$

$$\frac{\partial v}{\partial y} = (-1)(x^2+y^2+2x+1)^{-1} + y(x^2+y^2+2x+1)^{-2} \cdot 2y$$

$$\Rightarrow \frac{1}{\cancel{x^2+y^2+2x+1}} - \frac{2(x+a)(x+1)}{(x^2+y^2+2x+1)^2} = -\frac{1}{\cancel{x^2+y^2+2x+1}} + \frac{2y^2}{(x^2+y^2+2x+1)^2}$$

$$\Rightarrow \cancel{2} = \frac{\cancel{2}y^2 + \cancel{2}(x+a)(x+1)}{x^2+y^2+2x+1} \Rightarrow \cancel{x^2+y^2+2x+1} = \cancel{y^2} + (x+a)(x+1)$$

$$\Rightarrow (x+1)^{\cancel{2}} = (x+a)(\cancel{x+1}) \Rightarrow x+1 = x+a \Rightarrow \boxed{a=1}$$