Ε
Μ
Π

Άσκηση
Λυγισμός

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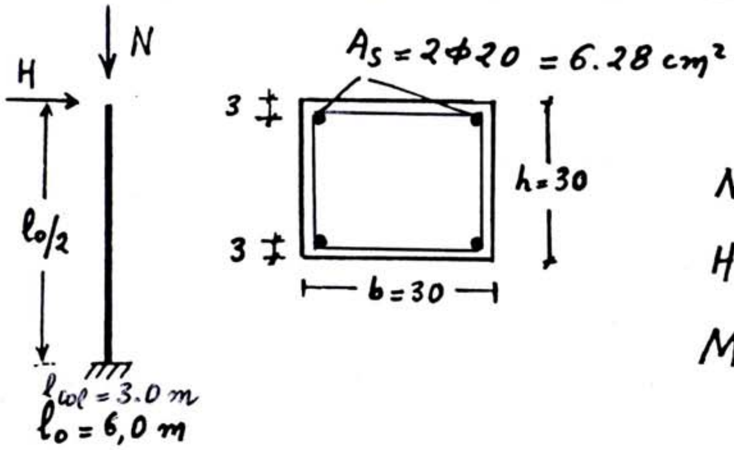
Εργαστήριο
Ωπλισμένου
Σκυροδέματος

Checking for buckling using M-1/r diagram

Έλεγχος σε λυγισμό με χρήση διαγράμματος M-1/r



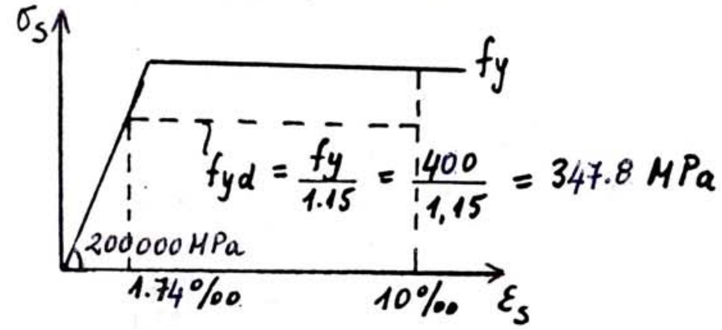
ΦΟΡΤΙΑ : Loads

$N_g = 200\ kN \qquad N_{sd} = 450\ kN$

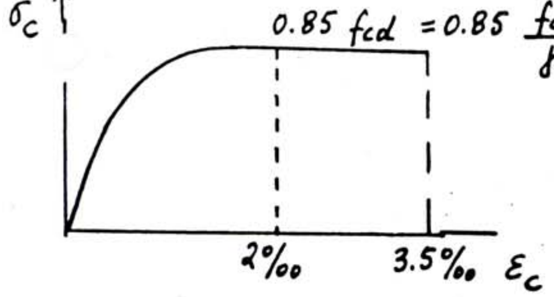
$H_g = 6{,}6\ kN \qquad H_{sd} = 15\ kN$

$M = 15\,.\,3 = 45\ kNm$

ΥΛΙΚΑ : materials — C30, S400 — $f_y = 400\ MPa$



$0.85\,f_{cd} = 0.85\,\frac{f_{ck}}{\gamma_c} = 0.85\,.\,\frac{30}{1.5} = 17\ MPa$



(§14.4.3) **Πρόσθετη εκκεντρότητα** : Imperfections:

ΕΚΩΣ $e_a = a\cdot\frac{l_0}{2} = 0.00577\,.\,\frac{600}{2} = 1.73 \simeq 2.0\ cm$

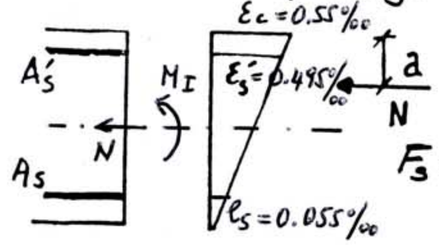
όπου $a = 1/100\sqrt{l_{col}}\ rad = 1/100\sqrt{3.0} = 0.00577$

στην κορυφή : $M = N_{sd} \times e_a = 450 \times 0{,}02 = 9\ kNm$ } Ροπές

στην βάση : $M = N_{sd} \times e_a + H_{sd}\cdot l = 9 + 15\,.\,3 = 54\ kNm$ } 1ης τάξεως

- Διάγραμμα ροπών - καμπυλοτήτων $(M - 1/r)$
Moment – Curvature Diagram

1) Τέλος σταδίου I (όλη η διατομή σε θλίψη)
End of stage I (all the section under compression)



$\varepsilon_c = 0.55‰$

$$F_c = \overset{\alpha}{0{,}25} \cdot \overset{x}{0{,}3} \cdot \overset{b}{0{,}3} \cdot \overset{0.85 f_{cd}}{17} = 0{,}382 \; MN = 382 \; KN$$

$$F_s + F_s' = 6{,}28 \cdot \frac{(0.055 + 0.495)}{1000} \cdot 20 = 0{,}069 = 69 \; KN$$

$$N_i = 451 \; KN \simeq 450 \; KN$$

$$E_s = 2{,}0 \cdot 10^7 \; t/m^2 = 20 \; MN/cm^2$$

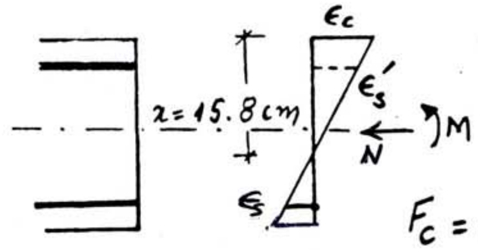
$$\gamma' = 0{,}345 \Rightarrow a = 0{,}3 \cdot 0{,}345 = 0{,}103 \; m$$

$$M_I \cong 380\,(0{,}15 - 0{,}103) + 70 \times 0{,}12 \cong 26 \; KNm$$

$$K = \frac{1}{r} = \frac{0{,}55}{0{,}30} \cdot 10^{-3} = 1{,}83 \cdot 10^{-3}$$

όπου $\alpha \rightarrow$ συντελεστής πληρώσεως } από διαγράμματα
$\gamma' \rightarrow$ συντελεστής Κ.Β.

2)



$\varepsilon_s = 1{,}0‰ \quad \varepsilon_c = 1{,}4‰ \quad \varepsilon_s' = 1{,}13‰$

$x = 15.8$ cm

$$F_c = \overset{\alpha}{0{,}54} \cdot \overset{b}{0{,}3} \cdot \overset{x}{0{,}158} \cdot \overset{0.85 f_{cd}}{17} \cdot 10^3 = 435 \; KN$$

$$F_s' = 6{,}28 \cdot 1{,}13 \cdot 20 \cdot 10^3 = 142 \; KN$$

$$F_s = 6{,}28 \cdot 1{,}0 \cdot 20 \cdot 10^3 = 126 \; KN$$

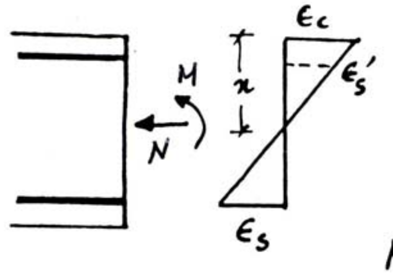
$$N_i = 451 \; KN \simeq 450$$

$$a = \overset{\gamma'}{0{,}36} \cdot 15{,}8 = 5{,}7 \; cm$$

$$M = 435\,(0{,}15 - 0{,}057) + (142 + 126) \cdot 0{,}12 = 72.61 \; KNm$$

$$\frac{1}{r} = \frac{1{,}0 + 1{,}4}{0{,}27} \cdot 10^{-3} = 8{,}9 \cdot 10^{-3}$$ (Σημείο A)
Point

3)



$\epsilon_s = 1{,}74‰ \quad \epsilon_c = 1{,}95‰ \quad x = 14{,}2\ cm$

$\epsilon_s' = 1{,}54‰$

$\sigma_s = f_{cd} = 347.8\ MPa$

$$F_c = \overset{\alpha}{0{,}66} \cdot\ \cdot 0{,}3 \cdot 0{,}142 \cdot 17 \cdot 10^3 = 478\ KN$$
$$F_s' = 6{,}28 \cdot 1{,}54 \cdot 20 \cdot 10^3 = 193\ KN$$
$$F_s = 6{,}28 \cdot 34.78 \cdot 10^3 = 218\ KN$$

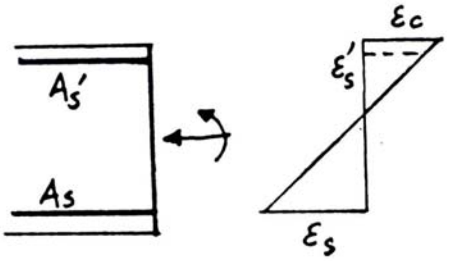
$N_i = 453 \simeq 450\ kN$

γ'

$$a = 0{,}375 \cdot 14{,}2 = 5{,}7\ cm$$
$$M = 478(0{,}15 - 0{,}057) + (193 + 218) \times 0{,}12 = 94\ KNm$$
$$\frac{1}{r} = \frac{1{,}95 + 1{,}74}{0{,}27} \cdot 10^{-3} = 13{,}7 \cdot 10^{-3}$$

(Σημείο Β)
Point

4) Αστοχία της διατομής λόγω αστοχίας σκυροδέματος
Failure of the compressive zone of concrete section



$\varepsilon_c = 3{,}5‰ \quad \varepsilon_s = 5‰ \quad \varepsilon_s' = 2{,}56‰$

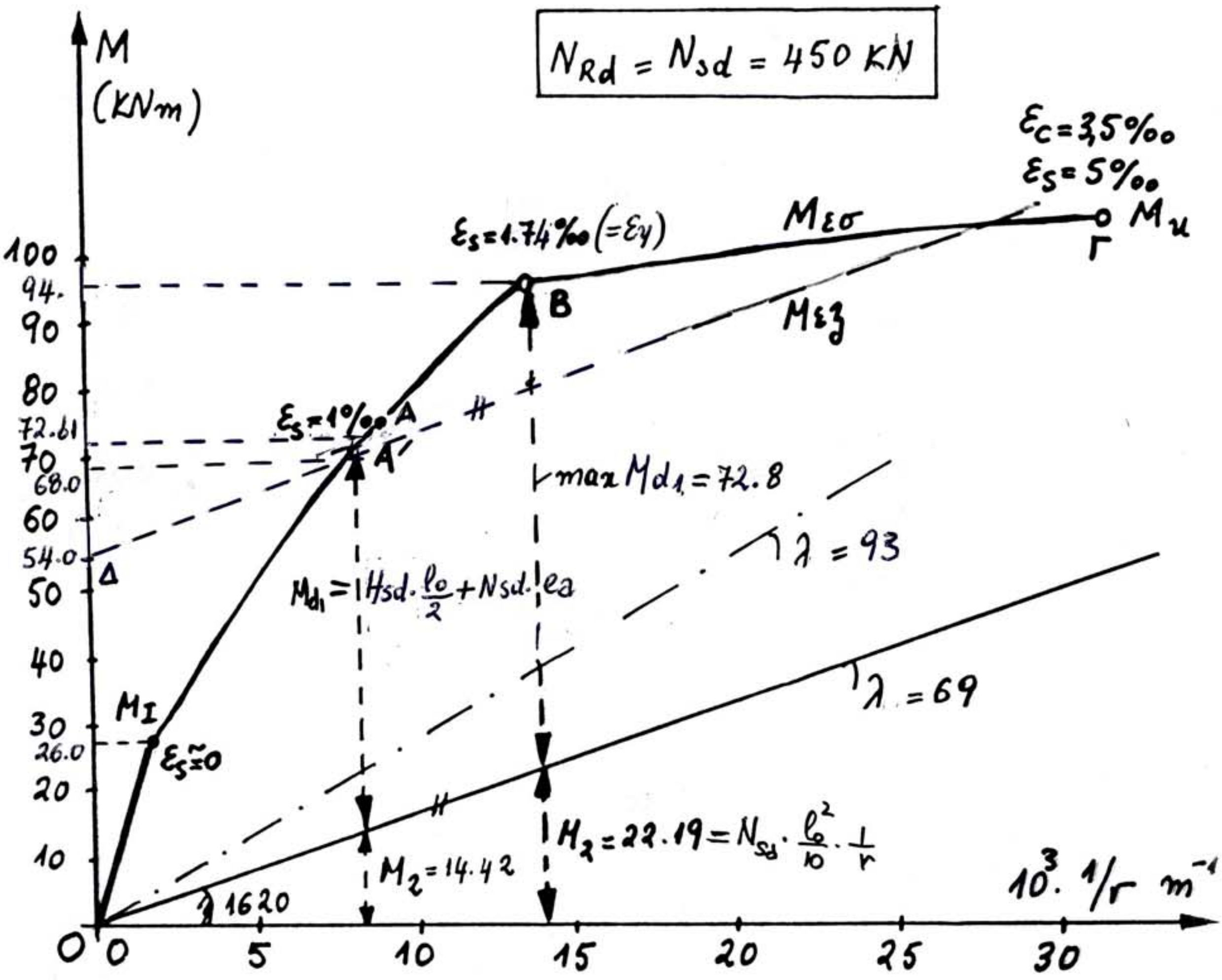
$x = 11{,}1\ cm$

$$F_c = \overset{\alpha}{0{,}81} \cdot 0{,}3 \cdot 0{,}11 \cdot 17 \cdot 10^3 = 454\ KN$$
$$F_s = F_s' = 6{,}28 \cdot 34.78 \cdot 10^3 = 218\ KN$$

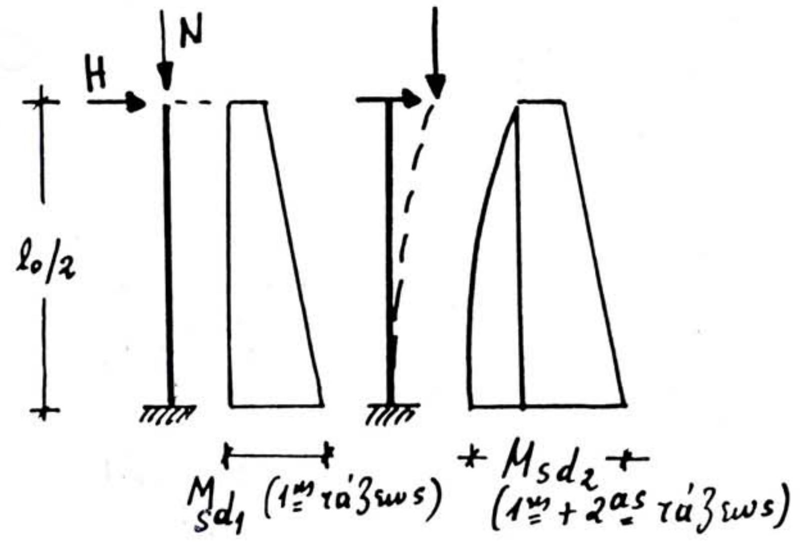
$N_i = 454\ KN \simeq 450$

$$a = 4{,}6\ cm$$
$$M_u = 454(0{,}15 - 0{,}046) + 218 \cdot 2 \cdot 0{,}12 = 99.5\ KNm$$
$$\frac{1}{r} = \frac{3{,}5 + 5{,}0}{0{,}27}\ 10^{-3} = 31.5 \cdot 10^{-3}$$

(Σημείο Γ)
Point



M (KNm)
$N_{Rd} = N_{sd} = 450$ KN
$\varepsilon_c = 3,5‰$
$\varepsilon_s = 5‰$
M_u
Γ
$\varepsilon_s = 1.74‰ (= \varepsilon_y)$
$M_{\varepsilon\sigma}$
B
$M_{\varepsilon\xi}$
$\varepsilon_s = 1‰$
A
A'
Δ
$\max M_{d1} = 72.8$
$\lambda = 93$
$M_{d1} = H_{sd} \cdot \frac{l_0}{2} + N_{sd} \cdot e_a$
$\lambda = 69$
M_I
$\varepsilon_s \simeq 0$
$M_2 = 22.19 = N_{sd} \cdot \frac{l_0^2}{10} \cdot \frac{1}{r}$
$M_2 = 14.42$
1620
$10^3 \cdot 1/r \ m^{-1}$
100
94.
90
80
72.61
70
68.0
60
54.0
50
40
30
26.0
20
10
O
0
5
10
15
20
25
30



N
H
$l_0/2$
M_{sd_1} (1ης τάξεως)
M_{sd_2}
(1ης + 2ας τάξεως)

<u>Ροπές 2ας τάξεως</u> : $M_2 = N_{sd} \cdot e_2$

όπου $e_2 = K_1 \cdot \frac{l_o^2}{10} \frac{1}{r}$ (εκκεντρότητα 2ας τάξεως) (14.18) ΕΚΩΣ

όπου l_o = μήκος λυγισμού

$1/r$ = καμπυλότητα στη βάση

$K_1 = \lambda/10 - 2.5$ εάν $25 \leq \lambda \leq 35$

$K_1 = 1$ $\lambda > 35$

όπου λ (λυγηρότητα) $= l_o/i$ (14.11) ΕΚΩΣ

όπου $i = \sqrt{I_c/A_c}$

εδώ $i = \sqrt{\frac{bh^3/12}{bh}} = h/\sqrt{12} = 30/\sqrt{12} = 8.66 \Rightarrow \lambda = \frac{600}{8.66} = 69$

άρα $K_1 = 1$ και $M_2 = N_{sd} \cdot \frac{l_o^2}{10} \frac{1}{r} = 450 \cdot \frac{36}{10} \frac{1}{r} = 1620 \frac{1}{r}$

<u>Εξωτερικές ροπές</u> : $M_{εξ} = M_{d1} + M_{d2} =$

$= \left(H_{sd} \frac{l_o}{2} + N_{sd} \cdot e_a\right) + N_{sd}\left(\frac{l_o^2}{10} \cdot \frac{1}{r}\right)$

$M_{d1} = 15 \times 3 + 450 \times 0,02 = 54.0$ KNm

Για $\frac{1}{r} = 0 \Rightarrow M_{εξ} = 54$ KNm $\Rightarrow$ σημείο Δ

Για $\frac{1}{r} = 8,9 \cdot 10^{-3} \Rightarrow M_{εξ} = 54 + 450 \cdot \frac{36}{10} \cdot 8,9 \cdot 10^{-3} =$

$= 54,0 + 14,42 = 68,42$ KNm $\Rightarrow$ σ. Α'

Η ευθεία $M_{εξ} \rightarrow$ Α'Δ, τέμνει την καμπύλη ΟΑΒΓ των $M_{εσ}$ και το υποστύλωμα είναι <u>ευσταθές</u>.

Διερεύνηση ως προς H_{sd}

$M_{Rd max} = 94$ KNm για $\frac{1}{r} = 13.7 \cdot 10^{-3}\ m^{-1}$

$M_2 = 1620 \cdot 13{,}7 \cdot 10^{-3} = 22{,}19$ KNm ∴ max $M_{sd} = 94 - 22{,}19 = 71.8$ KNm

Άρα : $H_{max} = \dfrac{\max M_{sd} - N_{sd}\, e_a}{l_o/2} = \dfrac{71.8 - 450 \cdot 0.02}{3} =$

$= \boxed{20.93\ KN = H_{max}}$

Διερεύνηση ως προς l_o

$M_{Rd max} = 94$ KNm $\quad M_{d1} = 54{,}0$ KNm $\Rightarrow \max M_2 = 94 - 54 = 40$ KNm

Για το δεδομένο υποστύλωμα : $\lambda_o = \dfrac{l_o}{\sqrt{\frac{I_c}{A_c}}} = \dfrac{}{\sqrt{\frac{30^4}{12 \cdot 30^2}}} = 69$

max λ_o = ; $\quad N_{sd} \dfrac{l_o^2}{10} \cdot 13{,}7 \cdot 10^{-3} = 94 - 54 = 40 \Rightarrow$

$l_o^2 = \dfrac{40 \cdot 10 \cdot 10^3}{450 \cdot 13{,}7} = 64.9 \Rightarrow l_{o\,max} = 8.055 \Rightarrow \boxed{l_{max} = 4.027\ m}$

$\rightsquigarrow \lambda_o = 93$

Επίλυση με τη βοήθεια πινάκων Μεγίστων ανηγμένων εξωτερικών ροπών υποστυλωμάτων (μέθοδος προτύπου υποστυλώματος)

Για να εφαρμοστεί η μέθοδος του προτύπου υποστυλώματος πρέπει :

$\lambda = \dfrac{l_o}{i} = \dfrac{l_o}{h/\sqrt{12}} = \dfrac{600}{30/\sqrt{12}} = 69.3 \;\; > \max\left(25, \dfrac{15}{\sqrt{\nu_d}}\right) = 33.5 \quad (14.13)$

$< 140 \quad (§\, 14.4.7.1)$

όπου $\nu_d = \dfrac{N_d}{A_c f_{cd}} = \dfrac{-450}{0.30 \times 0.30 \times 30\,000/1.5} = -0.25$

$d_1/h = 3.0/30 = 0.10$, S400

$l_o/h = 2 \times 3.00/0.30 = 20$

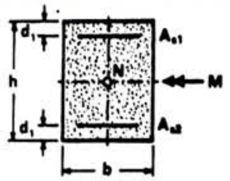
$\omega_{tot} = \dfrac{A_{s,tot}}{A_c} \cdot \dfrac{f_{yd}}{f_{cd}} = \dfrac{12.56}{900} \; \dfrac{400/1.15}{30/1{,}50} = 0.24$

$\xrightarrow{\text{πίνακες υπολογισμού}}$ για $\omega_{tot} = 0.2$ $\max \mu_{1d} = 0.130$

$\omega_{tot} = 0.4$ $\max \mu_{1,d} = 0.219$

άρα $\omega_{tot} = 0.24$ $\max \mu_{1,d} = 0.150$

$$\mu_{sd_1} = \frac{54.0}{0.30 \times 0.30^2 \times 30000/1.50} = 0.10 < \max \mu_{sd_1} = 0.150$$



$v_d = N_d / (A_c f_{cd})$

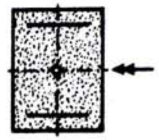
$\max\mu_{1d} = \max M_{1d} / (A_c h f_{cd})$

$\omega_{tot} = A_{s,tot} f_{yd} / (A_c f_{cd})$

Γιά κάθε C

$d_1 / h = 0.10$ S400

$A_{s1} = A_{s2} = \frac{1}{2} A_{s,tot}$



$-v_d =$

ΜΕΓΙΣΤΗ ΑΝΗΓΜΕΝΗ ΕΞΩΤΕΡΙΚΗ ΡΟΠΗ ΣΧΕΔΙΑΣΜΟΥ $\max\mu_{1d} =$

l_e/h	ω_{tot}	.00	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90
0	.0	.000	.044	.076	.096	.103	.099	.082	.054	.019											
	.2	.084	.123	.156	.176	.183	.168	.148	.123	.092	.057	.020									
	.4	.164	.202	.236	.256	.263	.242	.218	.192	.163	.131	.096	.059	.020							
	.6	.243	.282	.316	.336	.343	.319	.292	.263	.234	.203	.170	.135	.098	.060	.020					
	.8	.323	.362	.396	.416	.423	.397	.368	.337	.307	.276	.244	.210	.174	.137	.099	.060	.020			
	1.0	.403	.442	.476	.496	.503	.476	.445	.413	.382	.350	.317	.284	.249	.213	.176	.138	.099	.060	.020	
	1.2	.483	.522	.556	.576	.583	.554	.522	.490	.457	.425	.392	.358	.324	.289	.253	.215	.177	.139	.100	.060
	1.4	.563	.602	.636	.656	.663	.634	.600	.567	.534	.501	.467	.433	.399	.364	.329	.292	.254	.216	.178	.139
	1.6	.643	.682	.716	.736	.743	.713	.679	.645	.611	.577	.544	.509	.474	.439	.404	.368	.331	.294	.256	.217
5	.0	.000	.041	.071	.090	.097	.093	.076	.048	.014											
	.2	.084	.120	.151	.170	.177	.162	.141	.116	.085	.050	.015									
	.4	.164	.199	.231	.250	.257	.236	.211	.184	.156	.123	.088	.051	.016							
	.6	.243	.279	.311	.330	.337	.312	.284	.255	.226	.195	.161	.125	.089	.052	.016					
	.8	.323	.359	.391	.410	.417	.391	.360	.329	.298	.267	.234	.200	.164	.126	.089	.053	.016			
	1.0	.403	.438	.471	.490	.497	.469	.437	.404	.373	.340	.307	.273	.238	.202	.164	.127	.090	.053	.016	
	1.2	.483	.518	.551	.570	.577	.547	.514	.481	.448	.415	.381	.347	.312	.277	.240	.203	.165	.127	.090	.053
	1.4	.563	.598	.631	.650	.657	.627	.592	.558	.525	.490	.456	.422	.387	.352	.316	.279	.241	.203	.165	.128
	1.6	.643	.678	.711	.730	.737	.706	.671	.636	.601	.566	.532	.497	.462	.426	.391	.354	.317	.279	.241	.203
10	.0	.000	.037	.063	.079	.084	.078	.060	.031	.006											
	.2	.084	.114	.140	.159	.160	.142	.121	.096	.066	.035	.009									
	.4	.164	.192	.220	.239	.240	.216	.189	.161	.133	.101	.068	.038	.010							
	.6	.243	.271	.299	.319	.320	.292	.262	.231	.200	.169	.136	.103	.071	.040	.011					
	.8	.323	.351	.379	.399	.400	.370	.337	.304	.271	.238	.206	.172	.138	.105	.072	.041	.011			
	1.0	.403	.430	.459	.479	.480	.448	.413	.379	.344	.310	.276	.243	.208	.174	.139	.106	.074	.042	.011	
	1.2	.483	.510	.539	.559	.560	.527	.490	.454	.419	.384	.348	.313	.279	.245	.209	.175	.141	.108	.075	.043
	1.4	.563	.590	.619	.639	.640	.606	.568	.531	.495	.458	.422	.386	.351	.316	.281	.245	.210	.176	.142	.109
	1.6	.643	.670	.699	.719	.720	.685	.647	.609	.571	.534	.497	.460	.424	.388	.353	.318	.282	.246	.211	.177
15	.0	.000	.033	.054	.067	.069	.060	.039	.015												
	.2	.084	.108	.129	.142	.132	.115	.096	.072	.044	.020	.003									
	.4	.164	.185	.206	.222	.212	.183	.157	.132	.105	.075	.048	.025	.005							
	.6	.243	.264	.285	.302	.292	.258	.225	.195	.167	.138	.107	.079	.052	.028	.006					
	.8	.323	.342	.364	.382	.372	.336	.298	.263	.232	.202	.172	.140	.110	.081	.055	.030	.007			
	1.0	.403	.421	.444	.462	.452	.414	.374	.336	.301	.268	.237	.206	.173	.142	.112	.084	.057	.031	.007	
	1.2	.483	.501	.523	.542	.532	.492	.451	.410	.372	.337	.304	.272	.240	.207	.175	.144	.115	.086	.058	.033
	1.4	.563	.580	.603	.622	.612	.571	.528	.486	.446	.409	.373	.339	.306	.274	.241	.208	.177	.146	.116	.087
	1.6	.643	.660	.683	.702	.692	.650	.606	.563	.521	.482	.445	.409	.375	.341	.308	.275	.241	.209	.178	.147

$-\nu_d=$

ΜΕΓΙΣΤΗ ΑΝΗΓΜΕΝΗ ΕΞΩΤΕΡΙΚΗ ΡΟΠΗ ΣΧΕΔΙΑΣΜΟΥ $max\mu_{1d}=$

l_e/h	ω_{tot}	.00	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90
20	.0	.000	.030	.046	.054	.052	.040	.017	.001												
	.2	.084	.102	.116	.119	.103	.089	.071	.046	.023	.005										
	.4	.164	.178	.193	.199	.174	.147	.125	.102	.076	.050	.027	.010	.000							
	.6	.243	.255	.270	.279	.253	.216	.185	.159	.134	.106	.077	.052	.031	.013	.002					
	.8	.323	.333	.349	.359	.333	.290	.253	.221	.193	.165	.136	.106	.079	.054	.033	.015	.003			
	1.0	.403	.412	.427	.439	.413	.366	.325	.288	.256	.226	.196	.166	.136	.107	.081	.056	.036	.018	.003	
	1.2	.483	.491	.506	.519	.493	.444	.399	.359	.322	.289	.258	.228	.197	.165	.136	.107	.082	.058	.037	.019
	1.4	.563	.570	.586	.599	.573	.522	.475	.431	.392	.356	.323	.291	.259	.228	.195	.165	.136	.109	.083	.059
	1.6	.643	.650	.665	.679	.653	.601	.551	.506	.464	.425	.390	.355	.322	.290	.258	.226	.194	.165	.135	.109
25	.0	.000	.027	.039	.041	.034	.018	.000													
	.2	.084	.095	.099	.090	.073	.059	.041	.018	.000											
	.4	.164	.170	.175	.170	.137	.112	.090	.068	.042	.018	.001									
	.6	.243	.247	.252	.250	.210	.175	.146	.119	.094	.067	.040	.019	.004							
	.8	.323	.324	.330	.330	.286	.243	.208	.177	.148	.121	.092	.064	.040	.021	.006					
	1.0	.403	.402	.409	.410	.365	.316	.275	.240	.207	.176	.147	.117	.089	.062	.040	.022	.009	.001		
	1.2	.483	.481	.488	.490	.444	.391	.346	.306	.270	.236	.204	.173	.143	.113	.085	.061	.041	.024	.011	.002
	1.4	.563	.560	.567	.570	.523	.468	.419	.376	.335	.298	.264	.231	.200	.169	.137	.109	.084	.061	.041	.025
	1.6	.643	.639	.646	.650	.603	.545	.493	.447	.404	.364	.327	.292	.258	.226	.194	.163	.133	.107	.082	.060
30	.0	.000	.024	.032	.029	.018	.000														
	.2	.084	.086	.078	.058	.046	.031	.010													
	.4	.164	.161	.154	.134	.097	.070	.049	.027	.003											
	.6	.243	.237	.230	.213	.167	.129	.098	.071	.046	.020										
	.8	.323	.314	.308	.293	.240	.195	.156	.123	.094	.066	.039	.013								
	1.0	.403	.392	.386	.373	.316	.265	.221	.182	.147	.116	.087	.058	.031	.009						
	1.2	.483	.470	.465	.453	.393	.338	.289	.244	.206	.170	.138	.107	.077	.049	.025	.005				
	1.4	.563	.548	.544	.533	.471	.412	.360	.312	.268	.228	.193	.159	.127	.098	.068	.042	.020	.003		
	1.6	.643	.627	.623	.613	.550	.488	.433	.381	.334	.290	.251	.214	.180	.147	.117	.087	.060	.037	.017	.002
35	.0	.000	.021	.025	.018	.002															
	.2	.084	.075	.053	.039	.026	.007														
	.4	.164	.149	.128	.092	.055	.035	.014													
	.6	.243	.225	.204	.171	.117	.075	.047	.023	.000											
	.8	.323	.302	.282	.251	.189	.138	.095	.061	.034	.007										
	1.0	.403	.380	.360	.331	.264	.206	.156	.114	.077	.046	.018									
	1.2	.483	.457	.438	.410	.340	.277	.222	.173	.130	.093	.060	.030	.002							
	1.4	.563	.536	.517	.490	.417	.350	.291	.237	.189	.146	.108	.074	.043	.014						
	1.6	.643	.614	.596	.570	.495	.425	.361	.304	.251	.204	.162	.123	.088	.056	.026	.000				
40	.0	.000	.018	.019	.007																
	.2	.084	.063	.035	.024	.008															
	.4	.164	.136	.099	.049	.029	.009														
	.6	.243	.211	.174	.122	.062	.034	.011													
	.8	.323	.288	.251	.202	.131	.072	.038	.014												
	1.0	.403	.365	.329	.282	.204	.137	.082	.043	.016											
	1.2	.483	.443	.407	.361	.279	.207	.144	.090	.048	.019										
	1.4	.563	.521	.486	.441	.355	.279	.211	.150	.098	.053	.023									
	1.6	.643	.600	.565	.521	.433	.352	.279	.215	.155	.104	.059	.026								