6. WAVE SPEED AND LENGTH COMPUTATIONS W.W. Massie

6.1. Introduction

For intermediate water depths ($\frac{\lambda_o}{20} < h < \frac{\lambda_o}{4}$), there is no simple, direct means of determining the wave length or other related parameters given only the wave period. Two methods are presented here, both are derived from the non-linear equation for wave speed, equation 5.05.

6.2. Iteration Method

Recall equation 5.05,

$$c = \sqrt{\frac{g}{k} \tanh kh} = \frac{\lambda}{T} \qquad (5.05) \quad (6.01)$$

in which c is the wave phase velocity

g is the acceleration of gravity

k is the wave number = $\frac{2\pi}{\lambda}$

h is the water depth

λ is the wave length

T is the wave period.

Substituting various definitions from chapter 5 into equation 6.01 yields:

$$\lambda = \lambda_o \tanh \frac{2\pi h}{\lambda} \qquad (6.02)$$

Since λ, the unknown, cannot be isolated on one side of this equation, a direct solution is impossible. Iterative solution schemes *are* possible. In fact most any iteration will eventually lead to the correct answer since the equation has only one solution for given values of λ_o and h.

One simple but rather inefficient iteration is to resubstitute successive answers from 6.02 (starting with $\lambda = \lambda_o$) into the right hand side of the equation. Thus:

$$\lambda_{i+1} = \lambda_o \tanh \frac{2\pi h}{\lambda_i} \qquad (6.03)$$

where i = 0, 1, 2, ...

A much more efficient iteration is the following:

$$\lambda_{2i+1} = \lambda_o \tanh \frac{2\pi h}{\lambda_{2i}}$$

$$\lambda_{2i+2} = \frac{2\lambda_{2i+1} + \lambda_{2i}}{3} \qquad (6.04)$$

$$i = 0, 1, 2, \ldots\ldots$$

While the algorithm is a bit more complex, it reduces the number of iterations considerably (three or four are usually more than sufficient) and can still be executed on many of the small pocket electronic calculators.

A direct technique attributed to Eckert (unpublished) which usually gives answers correct to within about 5 percent is simply:

$$\lambda = \lambda_o \sqrt{\tanh \frac{2\pi h}{\lambda_o}} \qquad (6.05)$$

Table 6.1 compares the results of these schemes.

Table 6.1. Wave length iterations

T = 19 seconds, h = 50 meters

	eqn. 6.03	eqn. 6.04
i	λ_i (m)	λ_{2i+2} (m)
0	563.8	378.1
1	285.2	382.0
2	451.6	381.6
3	339.2	381.6
4	410.9	
5	362.9	
6	394.2	
7	373.4	
8	387.0	
9	378.0	
10	384.0	
11	380.1	
12	382.6	
13	380.9	
14	382.0	
15	381.3	
16	381.8	
17	381.5	

The superiority of the second iteration scheme is obvious. For comparison purposes, equation 6.05 yields λ = 401.0 which is off by 5.1%.

Obviously, now that the wave length has been determined all of the other related parameters can be easily evaluated.

6.3. Use of Tables

The computations outlined in the previous section were often cumbersome to carry out by hand. For this reason an alternative was developed in the form of a set of tables. By dividing both sides of equation 6.02 into h and doing a bit of algebra:

$$\frac{h}{\lambda_o} = \frac{h}{\lambda} \tanh \frac{2\pi h}{\lambda} \qquad (6.06)$$

in which $\frac{h}{\lambda_o}$ has been conveniently expressed in terms of $\frac{h}{\lambda}$. Thus, by choosing various values of $\frac{h}{\lambda}$, corresponding values of $\frac{h}{\lambda_o}$ can be computed directly and tabulated. Interpolation in this table working either toward values of $\frac{h}{\lambda_o}$ or toward values of $\frac{h}{\lambda}$ is all that is necessary to determine the wave length.

Wiegel (1954) worked out such a table. It is also published in his book *Oceanographical Engineering* (1964) and in the *Shore Protection Manual* (1973). An abbreviated version of this table is included here as table 6.2.

As an example, the previous iteration schemes can be checked. T = 19 sec. and h = 50 m yields λ_o = 563.80 m, and $\frac{h}{\lambda_o}$ = 0.0887. Interpolating in Wiegel (1964) yields $\frac{h}{\lambda}$ = 0.1310 and λ = 381.6 which compares rather favorably to the earlier calculation.

TABLE 6.2. SINUSOIDAL WAVE FUNCTIONS

$\frac{h}{\lambda_o}$	tanh kh	$\frac{h}{\lambda}$	kh	sinh kh	cosh kh	$\frac{H}{H_o'}$
0.000	0.000	0.0000	0.000	0.000	1.00	∞
002	112	0179	112	113	01	2.12
004	158	0253	159	160	01	1.79
006	193	0311	195	197	02	62
008	222	0360	226	228	03	51
0.010	0.248	0.0403	0.253	0.256	1.03	1.43
015	302	0496	312	317	05	31
020	347	0576	362	370	07	23
025	386	0648	407	418	08	17
0.030	0.420	0.0713	0.448	0.463	1.10	1.13
035	452	0775	487	506	12	09
040	480	0833	523	548	14	06
045	507	0888	558	588	16	04
0.050	0.531	0.0942	0.592	0.627	1.18	1.02
055	554	0993	624	665	20	1.01
060	575	104	655	703	22	0.993
065	595	109	686	741	24	981
070	614	114	716	779	27	971
0.075	0.632	0.119	0.745	0.816	1.29	0.962
080	649	123	774	854	31	955
085	665	128	803	892	34	948
090	681	132	831	929	37	942
095	695	137	858	0.968	39	937
0.10	0.709	0.141	0.886	1.01	1.42	0.933
11	735	150	940	08	48	926
12	759	158	0.994	17	54	920
13	780	167	1.05	25	60	917
14	800	175	10	33	67	915
0.15	0.818	0.183	1.15	1.42	1.74	0.913
16	835	192	20	52	82	913
17	850	200	26	61	90	913
18	864	208	31	72	1.99	914
19	877	217	36	82	2.08	916
0.20	0.888	0.225	1.41	1.94	2.18	0.918

$\frac{h}{\lambda_o}$	tanh kh	$\frac{h}{\lambda}$	kh	sinh kh	cosh kh	$\frac{H}{H_o'}$
0.20	0.888	0.225	1.41	1.94	2.18	0.918
21	899	234	47	2.05	28	920
22	909	242	52	18	40	923
23	918	251	57	31	52	926
24	926	259	63	45	65	929
0.25	0.933	0.268	1.68	2.60	2.78	0.932
26	940	277	74	75	2.93	936
27	946	285	79	2.92	3.09	939
28	952	294	85	3.10	25	942
29	957	303	90	28	43	946
0.30	0.961	0.312	1.96	3.48	3.62	0.949
31	965	321	2.02	69	3.83	952
32	969	330	08	3.92	4.05	955
33	972	339	13	4.16	28	958
34	975	349	19	41	53	961
0.35	0.978	0.358	2.25	4.68	4.79	0.964
36	980	367	31	4.97	5.07	967
37	983	377	37	5.28	37	969
38	984	386	43	61	5.70	972
39	986	395	48	5.96	6.04	974
0.40	0.988	0.405	2.54	6.33	6.41	0.976
41	989	415	60	6.72	6.80	978
42	990	424	66	7.15	7.22	980
43	991	434	73	7.60	7.66	982
44	992	443	79	8.07	8.14	983
0.45	0.993	0.453	2.85	8.59	8.64	0.985
46	994	463	91	9.13	9.18	986
47	995	472	2.97	9.71	9.76	987
48	995	482	3.03	10.3	10.4	988
49	996	492	09	11.0	11.0	990
0.50	0.996	0.502	3.15	11.7	11.7	0.990
1.00	1.000	1.000	6.28	268	268	1.000
∞	1.000	∞	∞	∞	∞	1.000