$$u_{xx}(x,t) = u_t(x,t) \Rightarrow F\{u_{xx}(x,t)\} = F\left\{\frac{\partial u(x,t)}{\partial t}\right\} \Rightarrow$$

$$\int_{-\infty}^{\infty} e^{i\lambda x} u_{xx}(x,t)\, dx = \int_{-\infty}^{\infty} e^{i\lambda x} \frac{\partial u(x,t)}{\partial t}\, dx \Rightarrow$$

$$(-\lambda i)^2 \int_{-\infty}^{\infty} e^{i\lambda x} u(x,t)\, dx = \frac{\partial}{\partial t}\int_{-\infty}^{\infty} e^{i\lambda x} u(x,t)\, dx \Rightarrow$$

$$-\lambda^2 \hat{u}(\lambda,t) = \frac{\partial \hat{u}(\lambda,t)}{\partial t} \Rightarrow \hat{u}_t(\lambda,t) + \lambda^2 \hat{u}(\lambda,t) = 0 \Rightarrow$$

$$\left(e^{\lambda^2 t} \hat{u}(\lambda,t)\right)' = 0 \Rightarrow \boxed{\hat{u}(\lambda,t) = C(\lambda) \cdot e^{-\lambda^2 t}} \quad (1)$$

Όμως $u(x,0) = 3e^{-x^2} \xRightarrow{F} F\{u(x,0)\} = F\{3e^{-x^2}\} \Rightarrow$

$$\boxed{\hat{u}(x,0) = \int_{-\infty}^{\infty} e^{i\lambda x} 3e^{-x^2} dx.} \quad (2)$$

$$(1) \xRightarrow{t=0} C(\lambda) = \hat{u}(\lambda,0)$$

Άρα $\hat{u}(\lambda,t) = \hat{u}(\lambda,0)\, e^{-\lambda^2 t} \xRightarrow{F^{-1}} F^{-1}\{\hat{u}(\lambda,t)\} = F^{-1}\{\hat{u}(\lambda,0)\, e^{-\lambda^2 t}\} \Rightarrow$

$$\boxed{u(x,t) = \int_{-\infty}^{\infty} e^{-\lambda x} \hat{u}(\lambda,0) \cdot e^{-\lambda^2 t}\, d\lambda}$$

όπου $\hat{u}(\lambda,0) = \int_{-\infty}^{\infty} e^{i\lambda x} \cdot 3e^{-x^2} dx$