$$z = f(\underbrace{x+\varphi(y)}_{u}) \qquad \varphi(y) \Rightarrow \frac{\partial \varphi}{\partial x} = 0 \;, \quad u = x+\varphi(y) \Rightarrow \begin{array}{l} \dfrac{\partial u}{\partial x} = 1 \\ \dfrac{\partial u}{\partial y} = \varphi' \end{array}$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot \cancel{\frac{\partial u}{\partial x}}^{\nearrow 1} = \frac{\partial z}{\partial u}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot \cancel{\frac{\partial u}{\partial y}}^{\nearrow \varphi'} = \frac{\partial z}{\partial u}\cdot \varphi'$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial\left(\frac{\partial z}{\partial x}\right)}{\partial x} = \frac{\partial\left(\frac{\partial z}{\partial u}\right)}{\partial x} = \frac{\partial\left(\frac{\partial z}{\partial u}\right)}{\partial u}\cdot \cancel{\frac{\partial u}{\partial x}}^{\nearrow 1} = \frac{\partial^2 z}{\partial u^2}$$

$$\frac{\partial z^2}{\partial x \partial y} = \frac{\partial\left(\frac{\partial z}{\partial x}\right)}{\partial y} = \frac{\partial\left(\frac{\partial z}{\partial u}\right)}{\partial y} = \frac{\partial\left(\frac{\partial z}{\partial u}\right)}{\partial u}\cdot \cancel{\frac{\partial u}{\partial y}}^{\nearrow \varphi'} = \frac{\partial^2 z}{\partial u^2}\cdot \varphi'$$

$$\left.\begin{array}{l} \dfrac{\partial z}{\partial x}\cdot\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial z}{\partial u}\cdot\dfrac{\partial^2 z}{\partial u^2}\cdot\varphi' \\[2ex] \dfrac{\partial z}{\partial y}\;\dfrac{\partial^2 z}{\partial x^2} = \dfrac{\partial z}{\partial u}\cdot\varphi'\cdot\dfrac{\partial^2 z}{\partial u^2} = \dfrac{\partial z}{\partial u}\;\dfrac{\partial^2 z}{\partial u^2}\cdot\varphi' \end{array}\right\} \Rightarrow \boxed{\frac{\partial z}{\partial x}\cdot\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial z}{\partial y}\;\frac{\partial^2 z}{\partial x^2}}$$