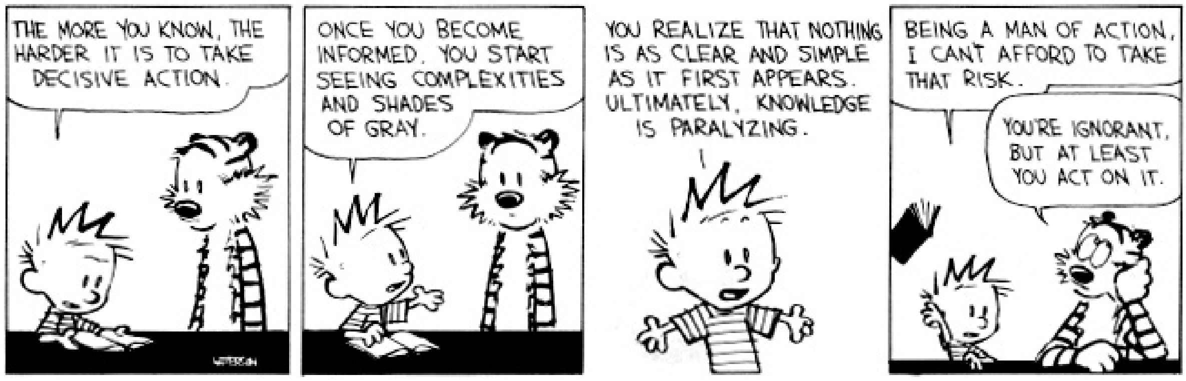
Homework 12. Chapters 13, 14, 14, 15.
Moments, torques, and static equilibrium



12.1 Concepts: Define and draw the moment of a force

Write the ***definition*** for the moment of force $\mathbf{F}^Q$ applied to point Q about point O. Include a ***sketch*** with *each* part of your definition clearly labeled.
Result:

$$\mathbf{M}^{\mathbf{F}^Q/O} \triangleq$$

12.2 Governing equations for static equilibrium

What **two** vector equations are used for determining **static equilibrium** of a system S?

-
-

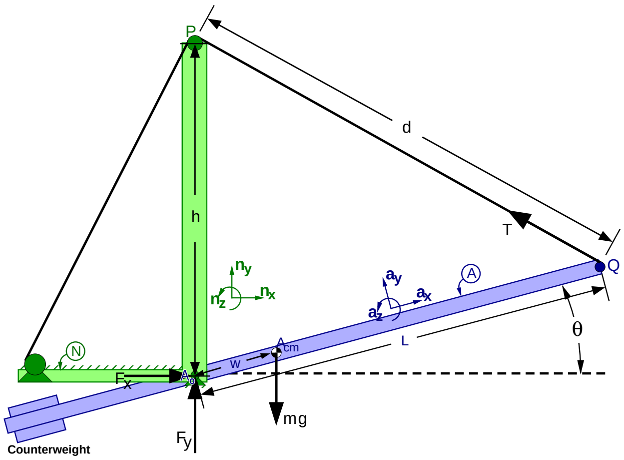
12.3 Static equilibrium of a draw-bridge.

The following figure shows a draw-bridge whose platform A (A includes the bridge's road-way and counter-weight) is supported by a frictionless revolute joint at point A_o and a light (massless) cable attached to point Q (the distal end of the platform). The cable runs over a pulley at point P and into a winch that is connected to ground N.

Right-handed sets of mutually perpendicular unit vectors $\mathbf{n}_i$ and $\mathbf{a}_i$ (i=x, y, z) are fixed in N and A, respectively, with $\mathbf{n}_x$ horizontally rights, $\mathbf{n}_y$ vertically upward, $\mathbf{a}_x$ directed from A_o to Q, and $\mathbf{n}_z = \mathbf{a}_z$ parallel to the revolute joint axis.

The following identifiers are useful in this analysis

Quantity	Identifier	Type
Distance between A_o and A_{cm}	w	constant
Distance between A_o and Q	L	constant
Distance between A_o and P	h	constant
Mass of A (includes roadway and counterweight)	m	constant
Local gravitational constant	g	constant
Angle between $\mathbf{n}_x$ and $\mathbf{a}_x$	θ	specified
Distance between P and Q	d	variable
Tension in cable	T	variable
$\mathbf{n}_x$ measure of reaction force on A at A_o	F_x	variable
$\mathbf{n}_y$ measure of reaction force on A at A_o	F_y	variable



${}^{a}R^{n}$	$\mathbf{n}_x$	$\mathbf{n}_y$	$\mathbf{n}_z$
$\mathbf{a}_x$			
$\mathbf{a}_y$			
$\mathbf{a}_z$			

(a) Complete the previous ${}^{a}R^{n}$ rotation table in terms of θ.

(b) Express $\mathbf{r}^{P/Q}$ (P's position vector from Q) as **efficiently as possible**.
Result:

$$\mathbf{r}^{P/Q} = \square\, \mathbf{n}_y + \square\, \mathbf{a}_x$$

(c) Form an expression for the distance d between P and Q in terms of h, L, and θ.
Result:

$$d = \square$$

(d) Verify the unit vector $\mathbf{u}$ directed from Q to P can be expressed in terms of L, d, h, and the unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$ and $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$ as shown below.

$$\boxed{\mathbf{u} = \frac{-L}{d}\,\mathbf{a}_x + \frac{h}{d}\,\mathbf{n}_y}$$

(e) Form $\mathbf{F}^{A}$, the resultant of all contact and distance forces on the road-way A.
Result:

$$\mathbf{F}^{A} = \square$$

(f) Knowing the draw-bridge's platform is in **static equilibrium**, solve for F_x and F_y in terms of T, m, g, h, L, d, and θ.
Result:

$$F_x = \square$$

$$F_y = \square$$

(g) Find the moment of all forces on A about A_o.
Result:

$$\mathbf{M}^{A/A_o} = \square$$

(h) Knowing the draw-bridge's platform is in **static equilibrium**, solve for T in terms of m, g, w, d, h, and L.
Result:

$$T = \square$$

(i) The maximum tension in the cable occurs when $\theta = \square^{\circ}$.

(j) The cable tension is **nonlinear** in (circle all applicable quantities)

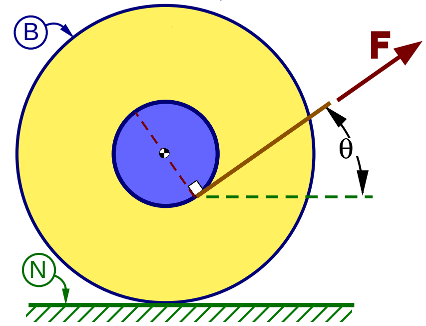
w $\quad L \quad h \quad m \quad g \quad d \quad \theta$

(k) Many draw-bridges have a counterweight because:

Physical explanation:

Mathematical explanation:

12.4 Static analysis of a wheel (Motivated by an old-fashioned penny farthing bicycle).

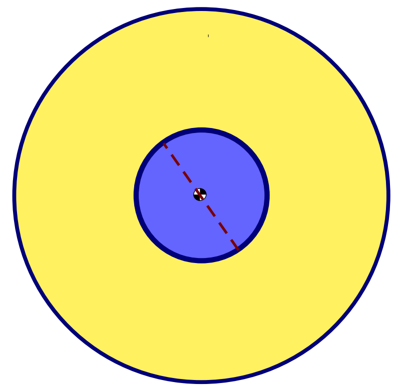


The figure to the right shows a force that pulls on a rope wrapped around the axle of a rigid wheel that is in contact with a **rough** flat horizontal plane N. The axle and wheel are rigidly connected and constitute a rigid body B.

Complete B's free-body diagram (below right). In the following table, **fully describe** any force measures that you introduce in your diagram. To facilitate your analysis, introduce helpful unit vectors, rotation matrices, points, etc.

Description of scalar quantities	Symbol
Local gravitational constant	g
Mass of B	m
Coefficient of static friction between B and N	μ_s
Radius of B's axle	r_A
Radius of B's wheel	r_W
Angle of the rope from the horizontal	θ
Measure of the force pulling on the rope	F_T

Complete description of additional force measures	Symbol



Assuming there is sufficient friction to prevent the wheel's sliding on N, calculate the angle θ_{static} (in terms of r_A and r_W) for B to be in **static equilibrium**.

Result:

$$\theta_{static} = $$

When $0° \leq \theta < \theta_{static}$, the wheel rolls **left/right**.

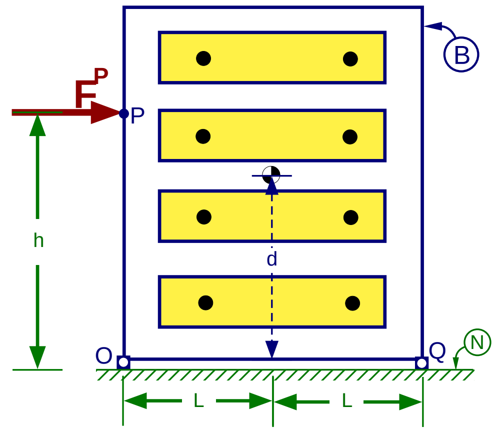
When $\theta_{static} < \theta \leq 90°$, the wheel rolls **left/right**.

Calculate the minimum μ_s for B to be in **static equilibrium** in terms of m, g, F, and θ.

Result:

$$\mu_{s_{\text{minimum}}} = $$

12.5 Bureau drawers with static friction.



The figure to the right shows a rigid bureau B in contact with a **rough** flat horizontal surface N at points O and Q of B.

The bureau has a mass m and its center of mass is elevated a distance e from the midpoint of O and Q (the midpoint is a distance L from both O and Q).

A person (not shown) is pushing the bureau horizontally right with a force of magnitude F^P applied to a point P of B that is located a height h above O.

The intent of this question is to investigate the role of static friction on the sliding and tipping of a bureau.

Answer the following questions in terms of h, L, d, m, g (the local gravitational acceleration), and μ_s (the coefficient of static friction between B and N).

- **Before doing any analysis**, use your intuition (guess) whether or not the **start** of the bureau tipping depends on (circle all that apply).

  $h \quad L \quad d \quad m \quad g \quad \mu_s$

- One way for the bureau to start tipping is for it to lose contact with N at point O. Mathematically, this means the normal and friction forces at O are $\mathbf{0}$. (N can push upward on B but cannot pull downward on B). In view of these facts, determine the minimum positive value of F^P to tip B.
  Result:
  $$F^P_{minimum} = \square$$

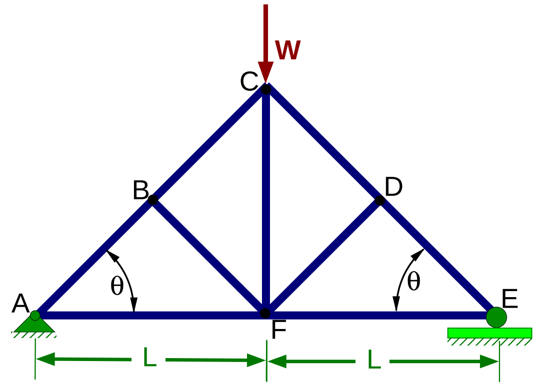
- Determine h_{tip}, the minimum height where a sufficiently strong push makes the bureau **start** to tip rather than slide.
  Result:
  $$h_{tip} = \square$$

- When $\mu_s \approx 0$, h_{tip} is **much smaller/smaller/equal to/larger/much larger** than L.
- In view of your static analysis, the **start** of the bureau tipping depends on (circle all that apply)

  $h \quad L \quad d \quad m \quad g \quad \mu_s$

12.6 Static analysis of a truss with roof load

The figure to the right is a ***planar truss*** of ***two-force members*** that is in **static equilibrium** and is attached to ground at point A by a pin-joint and point E by a pin-roller joint.

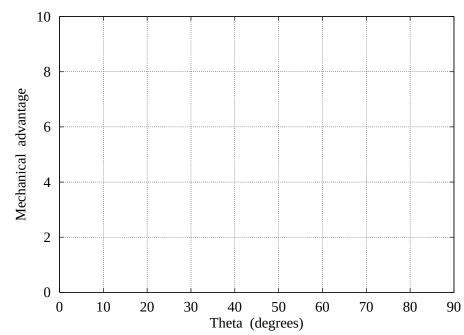


Complete the 2^{nd} column of the following table showing the magnitude of the force in each member as a function of L, θ and W (assume a positive value of W). In the 3^{rd} column, decide whether each member is in ***compression*** (forces on the member try to shorten it) or ***tension*** (forces on the member try to elongate it).

Member	Force magnitude	Compression or tension
AB		**Compression/Neutral/Tension**
AF		**Compression/Neutral/Tension**
BC		**Compression/Neutral/Tension**
BF		**Compression/Neutral/Tension**
CF		**Compression/Neutral/Tension**

- Consider a range of values for θ between $0°$ and $90°$.
  The minimum load in member AB occurs when $\theta =$ ____ °.
  The maximum load in member AB occurs when $\theta =$ ____ °.
  Increasing θ causes the load in member AB to **decrease/increase**.

- Machines can greatly magnify small forces. For example, a truss-like machine can be designed so that members BC and DC compress (pinch) an object at node C in the **horizontal** direction, each with a force of magnitude R. Determine a mathematical expression for the ***mechanical advantage*** of this truss-like machine (defined below) and plot it for $0° < \theta < 90°$.



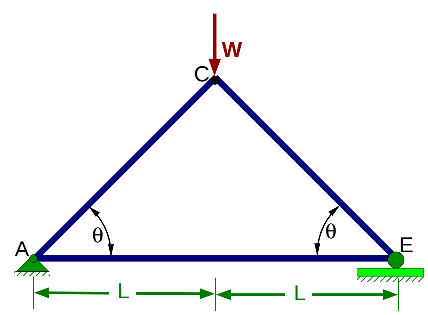
$$\text{Mechanical advantage} \triangleq \frac{R}{W} = \quad\square$$

- Members BF and DF carry no load. Why might an engineering add these members?

Euler's ***critical buckling load*** F_{buckling} for a long simply-supported column under an axial compressive force is

$$F_{\text{buckling}} = \frac{c}{L_{\text{column}}^2}$$

where L_{column} is the length of the column (member) and c is a constant that depends on the column's elastic modulus and area moment of inertia. Determine the value of θ (in degrees) that supports the maximum load W.



Result:

$$\theta = \quad\square\,°$$